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# THESIS

The Influence of Distance and Direction On Ground Combat Strength

by

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# The Influence of Distance and Direction On Ground Combat Strength

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**ABSTRACT**

This thesis describes the development from theorization to mathematical formulation of a ground combat model which includes the effects of range and orientation of fire and a valuation of mobility. The formulae of the model are then evaluated and expanded through the use of example calculations, which proceed from the most basic case to ever more complex situations. The result is a two-dimensional mapping of combat power in the front of a line of troops. The model and procedures allow a commander to evaluate tactical options when approaching an enemy, options which include different speeds, different directions and different troop placement. The model provides the foundation for two-sided measurement of the fire effects of attrition and suppression of enemy movement and return fire.

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# I. INTRODUCTION

Command and control is defined for United States military personnel as:

> The exercise of authority and direction by a properly designated commander over assigned forces in the accomplishment of the mission. Command and control functions are performed through an arrangement of personnel, equipment, communications, facilities, and procedures which are employed by a commander in planning, directing, coordinating and controlling forces and operations in the accomplishment of the mission [Ref. 1:p. 74].

The Soviet equivalent of command and control is referred to as troop control (upravleniye voyskami). Correlation of own and enemy forces has long been recognized as an essential part of Soviet troop control.

> Effectiveness of Troop/Naval-Force Control is considered by the Soviets to be a "Force Multiplier." The effectiveness of control is an important index of the combat capabilities of troops. Consequently, for estimating the combat capabilities and correlation of forces of sides it is necessary to make not only a qualitative determination of this index but also a quantitative one [Ref. 2:p. 3].

Operation analysts from the United States and other Western countries have attempted to develop quantitative methods for the correlation of forces. Various attempts at modeling combat have introduced formulae for computing and comparing quantitative indices relating own and enemy force strength to combat capability, usually called combat power. One method is to use an effectiveness index. In calculating the indices, all of the formulae isolate the two opposing forces, under the assumption that the measure of a force's value is independent of the opposing force. The major premise of this thesis is that quantitative indices of own and enemy forces must be calculated relative to each other, in order to estimate accurately the dynamic changes of combat power of two forces on a field of battle.

This thesis is based on the necessity that a correct mathematical model of own and enemy force strengths must include a dependent relationship between the two forces. A troop commander knows that in attacking an enemy from the rear his troops will not face as a great an opposing force strength as they would face in attacking the enemy's front. The change in the strength of the enemy does not result from a change in the quantity or quality of the enemy troops but, rather, results from the position, disposition and orientation of the opposing forces. Similarly, a commander knows that decreasing the distance between his own troops and the enemy troops causes an increase in the effective firepower which his troops can direct towards the enemy and a corresponding increase in the firepower which the enemy can direct against his troops. One should be able to incorporate these real world phenomena into a combat model and, in so doing, incorporate the value of maneuver in warfare. As used here, combat power is the capacity for combat activity toward the accomplishment of desired results.

It is through maneuverability that one force attempts to outflank the other, or an attacking force attempts to close quickly and ultimately overrun a defending force. Although it is through speed and agility of forces that one attempts to out-maneuver the opposition, velocity, itself, does not increase the strength of fire which one side can bring to bear upon the other, although it does reduce the effective strength of the enemy by introducing a moving target. This is not to say that the speed and agility of combat forces are not valued. Indeed, it is through greater speed and agility, in addition to intelligence/reconnaissance information, that one can take advantage of the orientation of enemy forces. The increase in effective strength, however, results from reducing the range to the enemy and from taking advantage of the orientation of the enemy forces. Quantifying the concepts of orientation-relative and distance-relative strengths are the subject of this thesis. While these concepts are not unique to ground combat, this thesis will limit its scope by attempting to create a ground combat model.

# II. DEVELOPING THE MODEL

## A. WHAT TO INCORPORATE

Theorists in the area of combat analysis maintain different views about what the best ground combat model should include. Disagreement even exists as to the dimensions that measure the strength of opposing sides. Theorists who simplify the outcome of battle as one consisting solely of casualties inflicted by both sides explain combat power by means of attrition models. In a basic attrition model, the index of strength of the opposing forces consists only of the number of shooters and the accuracy of their fire. Some of the more complex models include the use of equivalent shots, incorporating the size of projectiles; human factors, such as morale and leadership; environmental factors, such as weather and terrain, and still other combat factors. One of the most intricate of these models is the Quantified Judgment Model developed by Colonel T. N. Dupuy, U. S. Army (retired). Even in his complex model, the measure of strength of opposing sides is represented by the number of equivalent shots per second which can be fired by each side [Ref. 3]. The greatest objection to the attrition models appears to be their failure to include the aspects of position and mobility in the models.

Some analysts maintain that victory in ground combat is more frequently achieved by the side which maneuvers the best, as opposed to the side which attrits the other the most. Robert McQuie lists casualties or equipment loss as only the fourth most common reason for a force abandoning an attack or a defense, listing before it the reasons of envelopment, encirclement and/or penetration by the enemy; the withdrawal of an adjacent friendly unit; and having no reserves left [Ref. 4]. Captain Wayne P. Hughes, Jr., U. S. Navy (retired), has described combat power as the ability to maintain momentum while reducing enemy momentum. Momentum consists of forces and their rates of both movement and firing. Movement is the velocity multiplied by the number of movers and, in simple cases, firing activity is the firing rate multiplied by the number of shooters, although the computation of firing capacity can be as detailed as Dupuy's attrition model.

> Consider, as Von Neumann once did, the mongoose and the cobra: the mongoose wins by postures and movements before it strikes. The maneuvers are the battle, the strike behind the cobra's neck merely the consummation of what went before [Ref. 5:p. 12].

A momentum model helps describe, in a broader and more inclusive sense, the important components of ground combat, but they fail to achieve a dimensional

unity in measuring a force's combat power, in that each side is represented by both velocity, in meters per second, and firing rate, in shots per second.

While not solving this problem of dimensionality, another model incorporates yet another factor of combat in order to achieve a higher degree of descriptive accuracy of actual combat. This model incorporates suppression as a factor for describing combat. Supporters of this model, also stand opposed to attrition models of combat.

> An attrition orientation is a confusion of ends and means. Domination is the end, attrition in threat or fact is one means. An objective can be obtained after a wide variation in casualties and other destruction, from total to none. Nevertheless, lethality will always be involved, for lethality is the substance of combat power. Dominance is the result of superior combat power, or the perception in the loser of inferior combat power [Ref. 5:p. 35].

This model places emphasis on the effects which suppression has on the enemy in both reducing the number of shooters and in worsening the state of the human factors of the enemy forces.

> In summary, combatants seek to achieve their purpose by using lethality in united action to bend the will of the enemy: to dominate him. A winning force's combat power has the obvious effect of attrition, but of equal or greater potential importance, it also wins by the effects of suppression and demoralization on the enemy's state of mind and spirit [Ref. 5:p. 37].

The suppression model is similar in form to the momentum model, in that it portrays both firing capacity and movement capacity of a force. It, then, includes a factor of suppression, showing that suppression decreases both the firing capacity and movement capacity of a combat force. The suppression model falls short of being able to unite these concepts into an index or measure of the combat power exerted by two forces opposing each other on a battlefield.

In developing a quantitative model, one should not ignore these factors of firing capacity and movement capacity and how suppression diminishes this "momentum" of a combat force. Rather than attempting to unite these terms through some conversion formula, I shall maintain that the sole unit of measure must be that of effective shots per second, meaning that rate of delivery of well-aimed shots which strike targeted objects of nominal dimensions. Under ideal conditions, this would be the maximum attainable combat power of any shooter or, when multiplied by the number of shooters in the force, the maximum attainable combat power for that combat force. Combat, however, takes place in the real world, and other factors enter in combat to diminish this level of combat power--terrain, reduced visibility and enemy concealment being the most important.

Dupuy's QJM model incorporates many of these factors as historically based average effects. He reduces ordnance types to equivalent shots per second and includes muzzle velocities to enable a summation of different weapon types. He includes environmental factors, consisting of weather, season and terrain, and operational factors including of posture, mobility, vulnerability, fatigue, surprise and air superiority, and he conjectures human behavioral factors, consisting of leadership, training, experience, morale and manpower quality. The final result is a model which portrays firepower and its diminishment by these environmental and human factors, with the final output in units of "equivalent shots per second," which is very similar to the units of combat power in this thesis: hits per second against a benign target. We will define this to be effective shots per second and use it as our MOE. [Ref. 3]

## B. ACCOUNTING FOR MOBILITY

In his QJM model, Dupuy incorporates "mobility" as an operational variable. The force strength is then calculated by multiplying an operational lethality index by these variables [Ref. 3]. His approach, however, begs the question of how mobility affects friendly and enemy shooting power in positive and negative ways. Since mobility has the dimensions of velocity, e.g. meters/second, one cannot just add the two, and to multiply the two is to say that mobility is a force multiplier, which leads to the absurd conclusion that a dug-in, non-moving force has zero combat power in action.

In modeling combat power, the value of defilade can be taken into account without resorting to a measure of cubic meters of earth, by utilizing a factor which diminishes the hits per second of the shooters. The degree with which the factor diminishes the shooter's hitting rate depends upon the depth and composition of the defilade. In this same manner the value of mobility can be taken into account, without resorting to a measure of meters per second. In order to do this, we must be able to derive a relationship between mobility and combat power.

In discussing the Lanchester equations for modeling combat attrition, Bruce W. Fowler (among others) lists several assumptions as being implicitly contained in these equations. Two of the assumptions which apply to both Lanchester equations are: "2. All units are within weapon range of each other," and "3. Attrition rates are constant and known" [Ref. 6:p. 2]. The validity of these concepts must be addressed by any model which utilizes a measure of effective shots/s. Fowler goes on to state

> In summary, there is a rational basis for conjecture that the attrition rates are neither spatially nor temporally constant, in direct contradiction of rule 3. Further, if the attrition rates are not spatially constant, the validity of rule 2 becomes questionable [Ref. 6:p. 3].

It is in the knowledge that combat power is neither spatially nor temporally constant that we conclude that mobility must be incorporated into a ground combat model. In proceeding from this conclusion, rule 2 must still be addressed during the development of the model.

The question of how movement changes combat power, as measured in effective shots per second, can now be addressed. Movement affects targeting in several ways. First, a moving target provides more difficulty for a shooter than a stationary target. Second, depending upon the type of weapon system, movement which decreases the range to a target should cause an increase in the targeting effectiveness of the shooter. If we limit the discussion to small arms, we can definitely state that the targeting effectiveness increases as range decreases. Third, if the shooter, however, is moving, his own targeting effectiveness will decrease, in addition to a decrease in his firing rate. Fourth, movement away from the direction from which a defender expects and prepares for an attack will cause a decrease in the effectiveness of the defender. The attacker will have an initial advantage, but the degree of advantage depends upon how quickly the defender can re-orient his defensive posture.

Of these four phenomena, I maintain that the first and the third can be easily incorporated into a mathematical model by introducing two factors which diminish the effective shots per second, one based upon the relative velocity of the target and one based upon the shooters' velocity. In an equation, the effective firing rate can simply be multiplied by these two factors. The second and fourth phenomena will be much more difficult to incorporate, and it is with these two phenomena that we shall concern ourselves in our model. We shall begin with the incorporation of the second phenomenon, that of decreasing range causing an increase in targeting effectiveness.

## C. THE RANGE FACTOR

Two questions arise in dealing with range and combat power. The first question is how does range relate to effective shots per second, and the second is how we deal with the second assumption of the Lanchester equations, namely that all units are within weapon range of each other.

One suggestion for relating range to effective shots per second is made by Diane Brown and Alan Washburn in their hypothetical examples applied to a suppression model. They suggest an exponential increase of "lethality" (effective shots) as range decreases. In their examples, only small arms were considered. [Ref. 7]

An analysis of the range related performance of other weaponry was done in a range band analysis using STAR, performed by Dr. Sam Parry and LTC Edward P. Kelleher [Ref. 8]. The analysis involved defenders consisting of 12 XM1 tanks, eight IFV (TOW/Bushmaster), four ITV (TOV) and six DRAGON teams and attackers consisting of 30 T72 tanks and 9 BMP. Their results display a sharp increase in "kills/shot" as range decreases, but not as dramatic an exponential increase. In their analysis of kills per shot, they state

> The defender kills/shot tend to increase with decreasing range, whereas the attacker values are rather range independent. This fact is caused by increased attacker overkills at short ranges and the relative availability of targets, as well as the inherent defilade advantage [Ref. 8:p. 4].

In incorporating the range dependency, overkills will not be deleted from the model, since "effective shots" describe only shots which strike targeted objects, without addressing state of the object being targeted. Since we are restricting this model to one of small arms for simplicity in development, we can begin by looking at an exponential relationship between range and effective firepower. First, we take the maximum firing rate, designated by "q," of an M-16A2, which is 11.7 shots per second, assuming that the <u>effective</u> rate of fire will be a linear fraction of that rate. Next, we define that rate to be the maximum effective firepower as the range, designated by "r," approaches zero meters. Finally, we can plot out the exponential decay of the effective firepower as the range increases. Figure 1 shows the results.

Two things are wrong with these results. The first thing is that the effective firepower should approach zero near the maximum effective range of the weapon, instead of at as short a range as displayed in Figure 1. The second problem is one of dimensions. When we say that the effective firepower decays exponentially as the range increases, we mean that

$P \sim q * e^{-r}$.

The exponential term is raised to a power with the dimensions of meters, and this is incorrect. The exponential term must be raised to a dimensionless power, in order to preserve the dimensions of the equation. At this point, we are developing an equation for combat power with the dimensions of effective

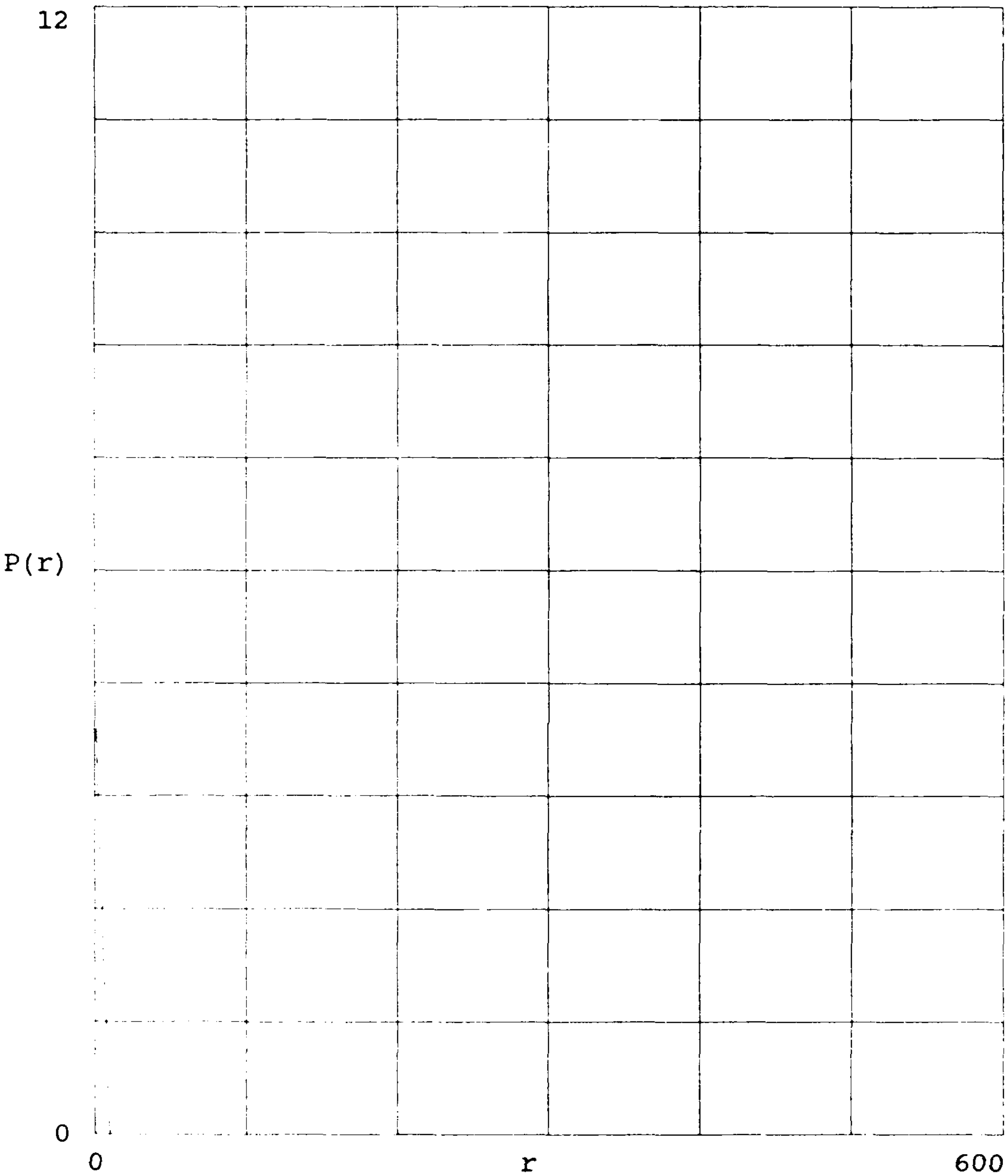


Figure 1--Plot of Effective Firepower $(P(r) \sim q*e^{-r})$
(effective shots/s) Versus Range (meters)

shots per second. In order to make the exponential power dimensionless, we must divide the range term by another term, designated by "k," which has the same dimensions as the range term, that is meters. Now, our equation for combat power looks like

$$P \sim q * e^{-\frac{r}{k}}.$$

This solves the problem of dimensions. We still have the problem of the combat power decaying too quickly.

We will, now, adjust the decay such that the effective shots per second approach zero at the maximum effective range. We have simultaneously solved this problem and have answered our second question concerning the relationship between combat power and range, that of how to account for all units being within weapon range of each other. Instead of having to concern ourselves with maps displaying units and their associated weapon's range arcs, this rule will be incorporated in the equation. Finding the value for "k" is done by letting the combat power approach zero when the range is equal to the maximum effective range. The result is that "k" is approximately equal to one-fourth of the maximum effective range. For computational simplicity, we will use one-fourth of the maximum effective range in all of our quantitative examples.

Applying our equation for combat power, with "k" equal to 150 meters, one-fourth of the maximum effective range of an M-

16A2 rifle, and "q" still equal to 11.7 shots per second, the maximum firing rate of the M-16A2, we can plot an example of our new relationship between combat power and range. Figure 2 displays this plot. Next, we can incorporate into our combat power equation the fourth phenomenon of spatially dynamic combat power, that of how movement away from the direction from which a defender expects and prepares for an attack will cause a decrease in the effectiveness of the defender.

## D. THE ORIENTATION FACTOR

The incorporation of the fourth phenomenon requires a relationship between combat power and the direction from which a unit is prepared for an attack, hereafter referred to as a unit's orientation, such that combat power is a maximum along the line of orientation, representing the direction directly in front of a unit. The combat power should fall off equally, as measured away from the axis, toward either flank, and should achieve a minimum along a line 180 degrees from the axis of orientation, representing the direction immediately behind a unit.

Such a relationship, as suggested by Sam Parry, can be represented by a cardioid, like the one shown in Figure 3 [Ref. 9]. In incorporating this into our combat power equation, the combat power "P" should be a maximum along the axis of orientation, where it will be equal to "q," the

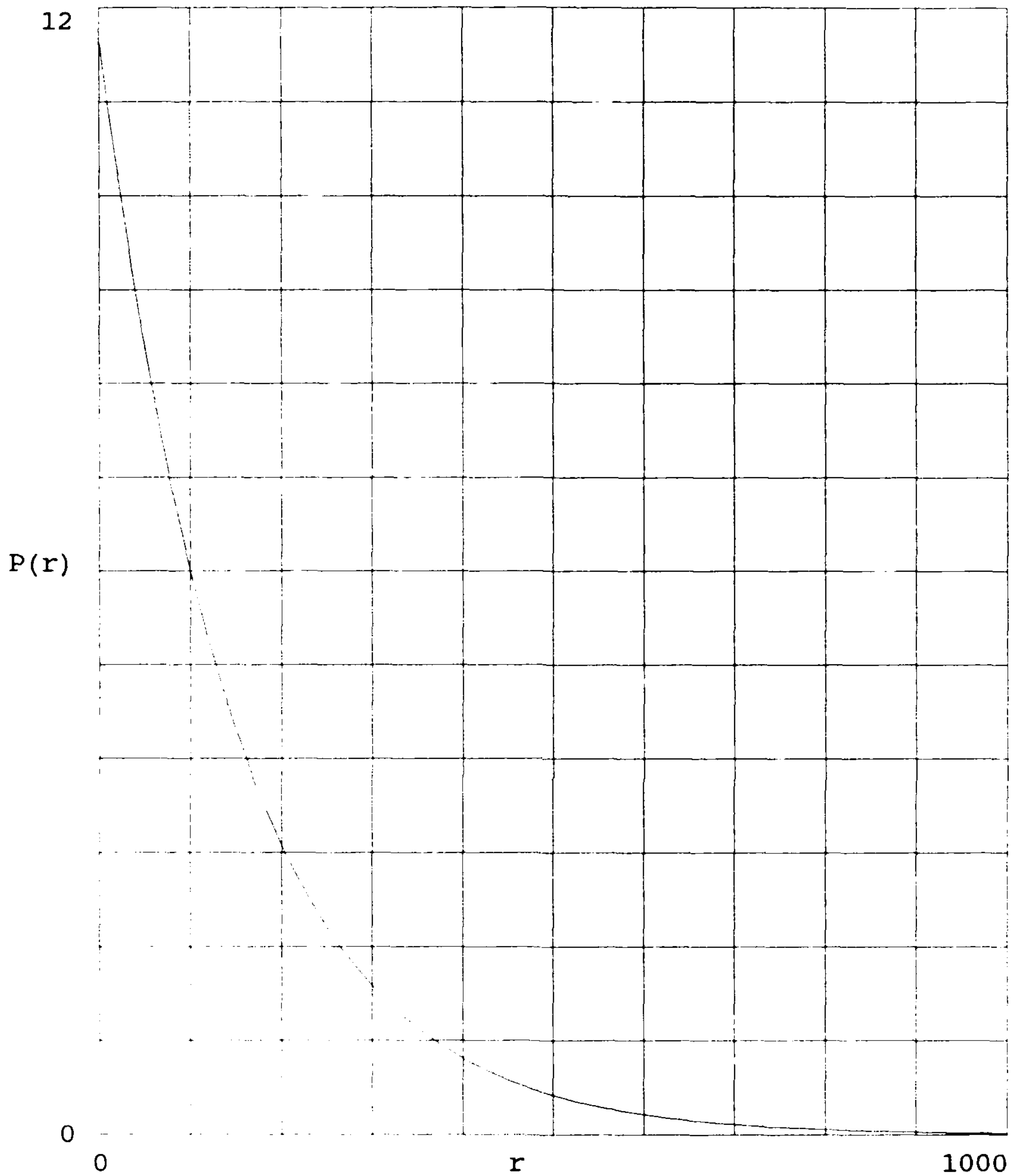


Figure 2--Plot of Effective Firepower ($P(r) = q*e^{-r/k}$) (effective shots/s) Versus Range (meters)

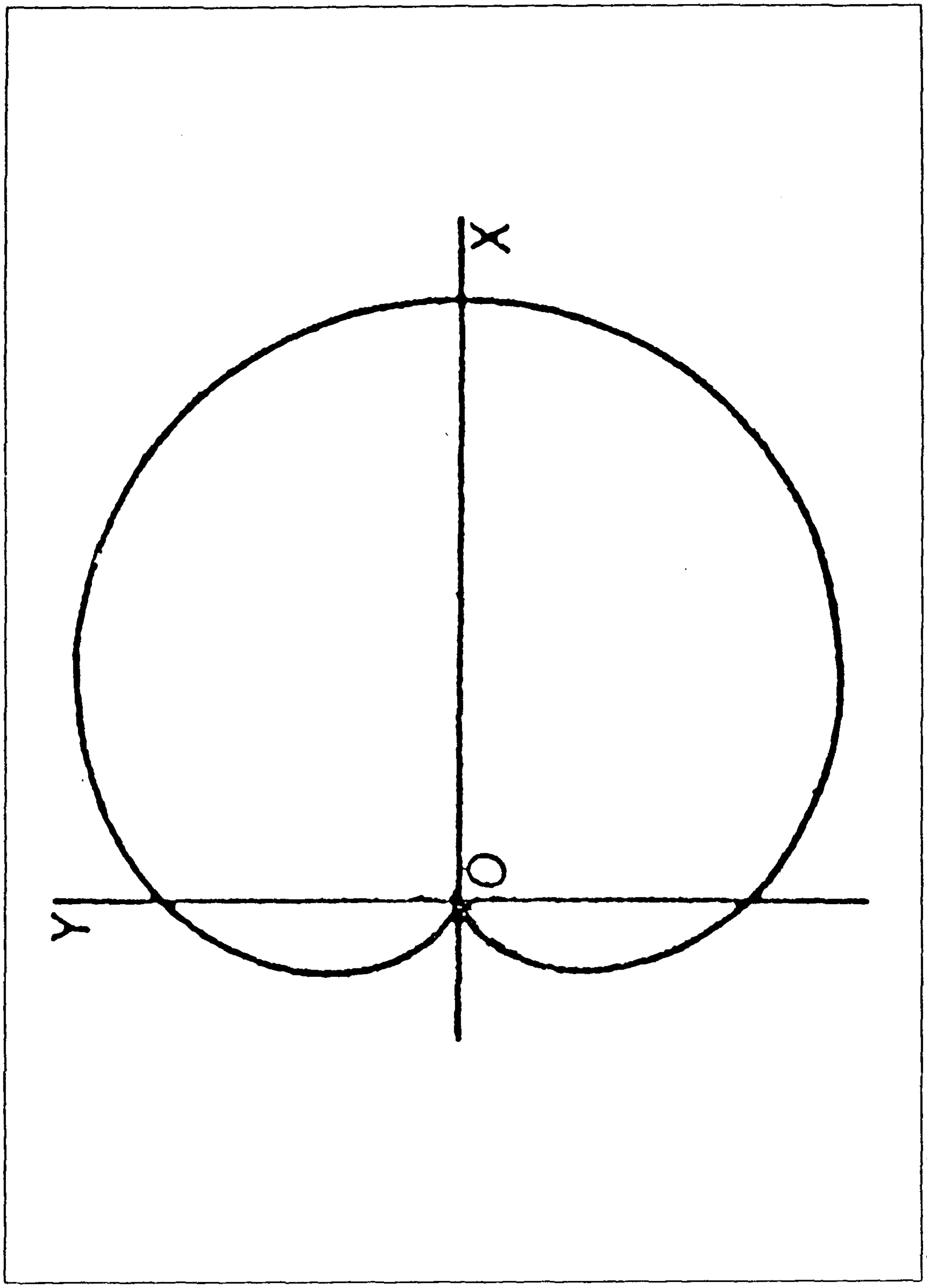


Figure 3--A Cardioid

maximum effective firing rate, ignoring for the moment the effects of range. Therefore, we may state that

$$P \sim q * 0.5 * (1 + \cos\theta),$$

where $\theta$ is the angle offset from the axis of orientation, over angles of from zero to plus or minus pi radians. Using this equation, the combat power does achieve a maximum of "q" directly in front of the unit and reaches a minimum of zero directly behind the unit. Figure 4 shows the plot of combat power versus $\theta$, with "q" equal to 11.7 shots per second.

Having addressed the problem of the spatially dynamic nature of combat power, it is necessary to, again, stress that combat power is, also, temporally dynamic. The equation above, relating combat power to orientation, describes the combat power at only one moment in time. Should either the firing rate or the orientation change, then the combat power will change. Similarly, the movement of the opponent, represented by a change in $\theta$, will, also, change the combat power. Finally, we need to combine our two spatial relationships into one equation.

## E. THE FINAL MODEL

Our final equation is simply the product of the combined terms from the two previously identified relationships. The result is

$$P = 0.5 * q * (1 + \cos\theta) * e^{-r/k} .$$

Given the shooter's maximum firing rate and orientation and

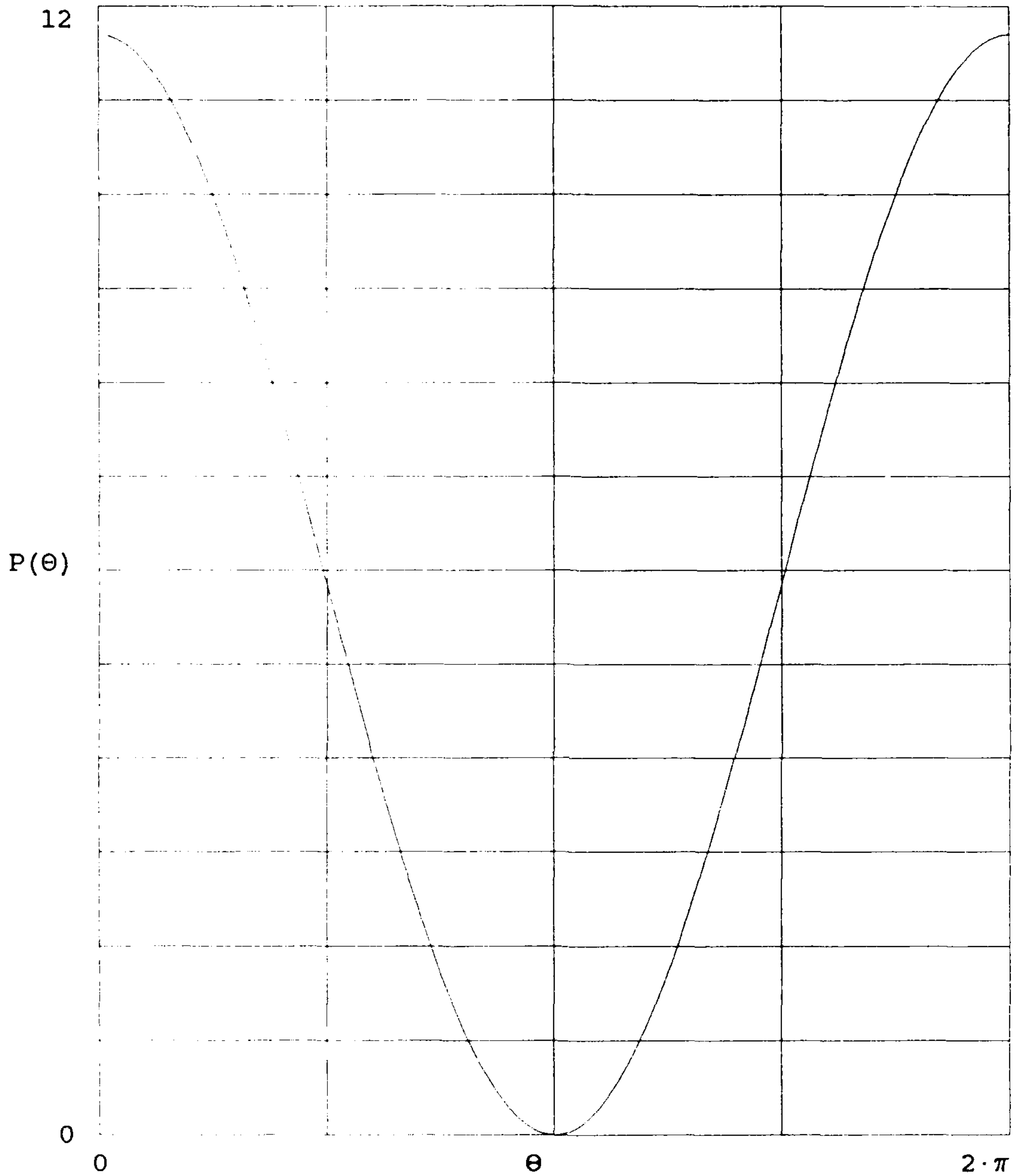


Figure 4--Plot of Effective Firepower (P(Θ) ˜ 0.5*q*(1 + cos(Θ))

(effective shots/s) Versus Range (meters) the opponent's position relative to the shooter, this equation can compute the combat power of the shooter. As the firing rate, orientation and position of the shooter and/or the position of the opponent change, the combat power will also change.

Uses of the model will be detailed in later chapters, but one example would be that, given a defender's location, his defensive posture (orientation) and an estimate of his firing rate, an attacking commander could use the combat power equation to compare and evaluate different options for an attack. In order to examine the uses of the model, we will develop example problems to which the equation can be applied. The examples will begin at a very basic level and will progressively more complex. The development will parallel the development common in physics classes concerning electrical charges and fields and the forces resulting from their interaction.

# III. EXAMINING THE MODEL

## A. A POINT FIREPOWER AGAINST A STATIONARY POINT TARGET

### 1. Formulation

Let the maximum firepower of a single rifleman be represented by "q" in terms of shots per second. In spatial dimensions, let each rifleman be represented as a single point. At an instant in time, each of these points of firepower has a direction of orientation associated with it. Over a length of time, the direction of orientation represents the direction in which the rifleman expects to apply his firepower. Each point firepower has a field of potential firepower associated with it, which can be described at any given point by

$$P = 0.5 * q * (1 + \cos\theta) * e^{-r/k}. \qquad (3\text{-}1)$$

where $\theta$ is the offset angle, with a range of values of zero to two-pi radians, representing the offset relative to the point firepower's direction of orientation, along which axis $\theta = 0$, "r" is the radial distance from the point firepower and "k" is a constant proportional to the maximum effective range of a rifleman, k = 1/4 maximum effective range. The dimensions of "k" are the same as those of "r" (Meters will be used in all examples.), and those of "P" are effective shots per second, as explained in Chapter II.

To find "P" for a group of point firepowers, calculate P(n) due to each firepower at the given point as if it were the only firepower present and add these separately calculated fields to find the resultant field "P" at this point. In equation form,

$$P = P_1 + P_2 + P_3 + \cdots . \qquad (3\text{-}2)$$

2. Example 1

Figure 5 shows two point firepowers, of equal magnitude and oriented in the same direction, placed a distance "d" apart. What is the magnitude of field "P" due to these firepowers at point "S," a distance "x" along the perpendicular bisector of the line joining the firepowers in the direction of orientation of the two firepowers?

Equation 3-2 gives

$$P := P_1 + P_2 ,$$

where from equation 3-1

$$P_1 := 0.5 \cdot q \cdot (1 + \cos\theta) \cdot e^{-\left[\frac{r}{k}\right]} \quad \text{and}$$

$$P_2 := 0.5 \cdot q \cdot (1 + \cos(2 \cdot \pi - \theta)) \cdot e^{-\left[\frac{r}{k}\right]} .$$

Substituting these values into the equation for "P" yields

$$P := 0.5 \cdot q \cdot (1 + \cos\theta) \cdot e^{-\left[\frac{r}{k}\right]} + 0.5 \cdot q \cdot (1 + \cos(2 \cdot \pi - \theta)) \cdot e^{-\left[\frac{r}{k}\right]} .$$

Simplifying gives us

$$P := q \cdot (1 + \cos\theta) \cdot e^{-\left[\frac{r}{k}\right]} .$$

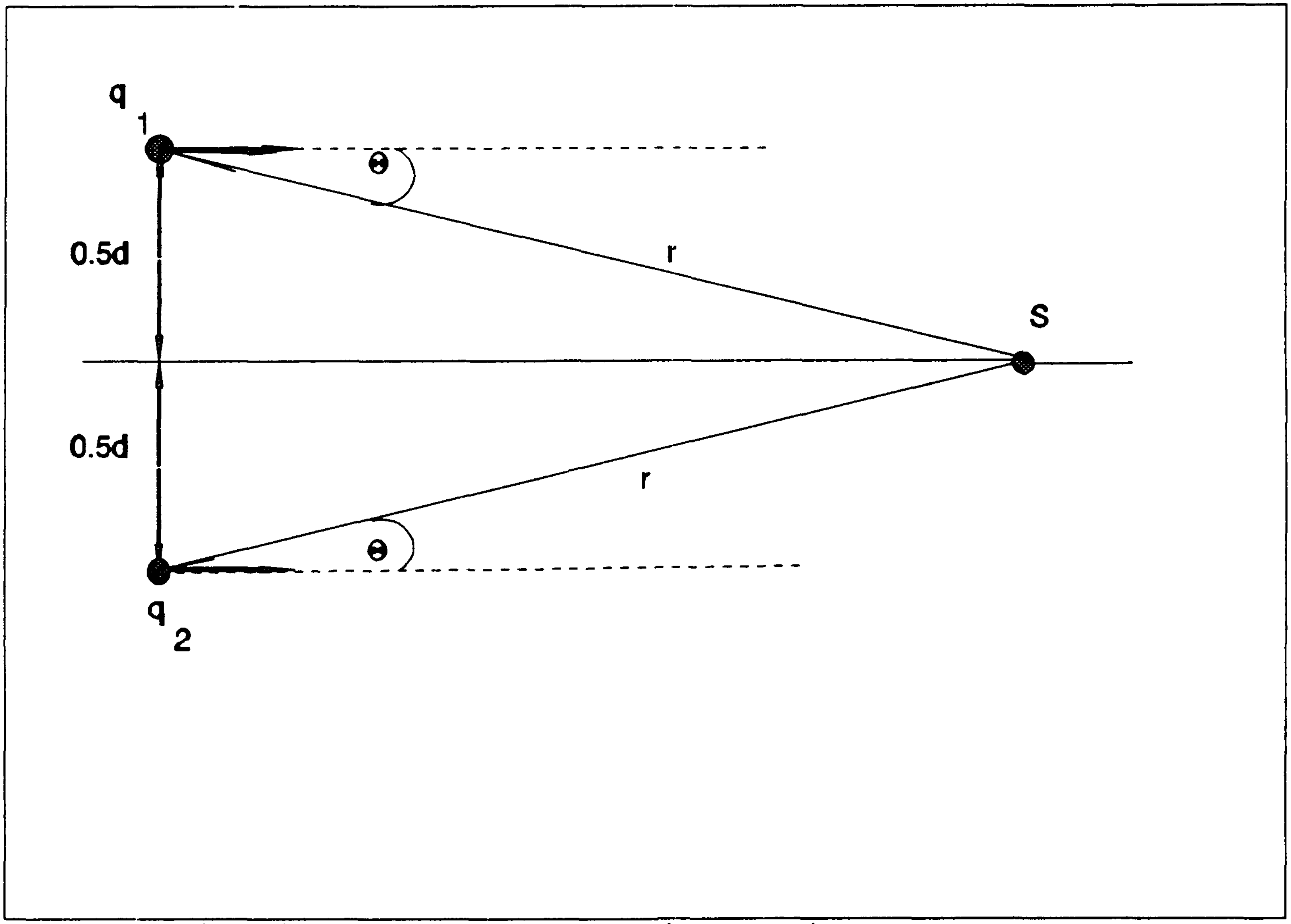


**Figure 5**--Point Firepowers Against a Stationary Point Target

3. Example 2

Figure 6 shows a point firepower $q_1 := 11.70$ shots/s placed 40 meters from a point firepower $q_2 := 9.17$ shots/s. Both firepowers are oriented in the same direction. The constants have the values $k_1 := 150$ m and $k_2 := 450$ m. What is the field "P" due to these firepowers at point "S," a distance of 200 meters forward of the firepowers, along their axis of orientation, and offset 60 meters to the "left" of $q_1$?

Looking at Figure 6, we see that $r_1$ and $r_2$ are the respective distances and $\theta_1$ and $\theta_2$ are the respective offset angles from $q_1$ and $q_2$ and that $y_1 := 60$ m, $d := 40$ m and $x := 200$ m. We can now calculate the missing variables.

$y_2 := y_1 + d$ ; $y_2 = 100$ m.

$$\theta_1 := \operatorname{atan}\left[\frac{y_1}{x}\right] \quad ; \quad \theta_1 = 0.29146 \quad \text{rad.}$$

$$\theta_2 := \operatorname{atan}\left[\frac{y_2}{x}\right] \quad ; \quad \theta_2 = 0.46365 \quad \text{rad.}$$

$$r_1 := \left[\frac{x}{\cos\left[\theta_1\right]}\right] \quad ; \quad r_1 = 208.80613 \quad \text{m.}$$

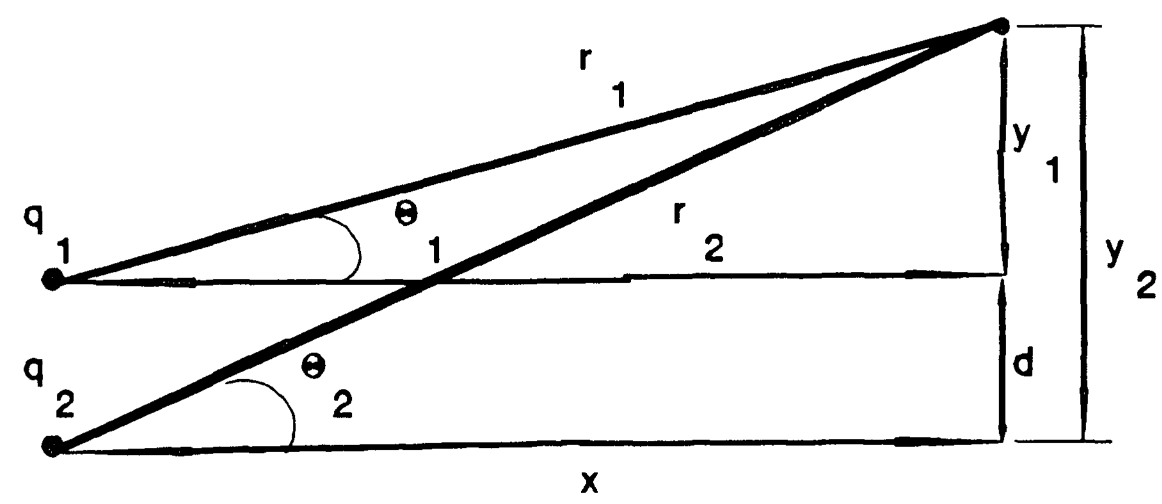


**Figure 6**--Unequal Point Firepowers Against a Stationary Point Firepower

$$r_2 := \left[\frac{x}{\cos\left[\theta_2\right]}\right] \quad ; \quad r_2 = 223.6068 \quad \text{m.}$$

From equation 3-1,

$$P_1 := 0.5 \cdot q_1 \cdot \left[1 + \cos\left[\theta_1\right]\right] \cdot e^{-\left[\frac{r_1}{k_1}\right]} \quad \text{and}$$

$$P_2 := 0.5 \cdot q_2 \cdot \left[1 + \cos\left[\theta_2\right]\right] \cdot e^{-\left[\frac{r_2}{k_2}\right]}$$

Substituting the previously computed variables into the equations for $P_1$ and $P_2$ gives us

$P_1 = 2.84691$ effective shots/s and

$P_2 = 5.28463$ effective shots/s.

Equation 3-2 gives

$$P := P_1 + P_2 \quad .$$

Substituting the values for $P_1$ and $P_2$ yields

$P = 8.13154$ effective shots/s.

The result displays the advantage, when there is a clear field of fire, of weapons with a longer maximum effective range, such as the advantage of an M-60 machine gun over an M-16A2 rifle in automatic, and this incorporates neither the difference in size and weight of the respective projectiles nor the difference in frequency of reloading. The values of $q_1$ and $k_1$ are the maximum firing rate and 1/4 the maximum effective range of an M-16A2. The values of $q_2$ and $k_2$ are the maximum firing rate and 1/4 the maximum effective range of an M-60. Despite having a smaller firing rate, a larger offset angle and a longer distance, the potential field at point "S" resulting from firepower $q_2$ is 1.86 times greater than that resulting from $q_1$. This is significant to note, since most models take into account only the size and weight of projectiles in distinguishing between an automatic rifle and a machine gun. A few models have incorporated muzzle velocity, but, again, the muzzle velocity of the M-16A2 is greater than that of the M-60.

## B. AN INFINITE LINE OF FIREPOWER AGAINST A STATIONARY POINT TARGET

### 1. Formulation

If the firepower distribution is a continuous one, the field which it sets up at any point "S" can be computed by dividing the charge into infinitesimal elements "dq." The field "dP" due to each element at the point in question is then calculated, treating the elements as point firepowers. The magnitude of "dP" is given by

$$dP := 0.5\cdot(1 + \cos\theta)\cdot e^{-\left[\frac{r}{k}\right]}\cdot dq \tag{3-3}$$

where "r" is the distance from the firepower element "dq" to the point "S." The resultant field at "S" is then found by integrating the field contributions due to all the firepower elements, that is,

$$P := \int_{-\infty}^{\infty} dP \tag{3-4}$$

### 2. Example 3

Figure 7 shows a section of an infinite line of firepower with an orientation parallel to the x-axis and on the positive direction and whose linear charge density (that is, the firepower per unit length, measured in shots/s*m) has the constant value $\lambda$. Calculate the magnitude of the field "P" at point "S" a distance "x" from the line.

The magnitude of the field contribution "dP" due to firepower element dq (= $\lambda$*dy) is, using equation 3-3, given by

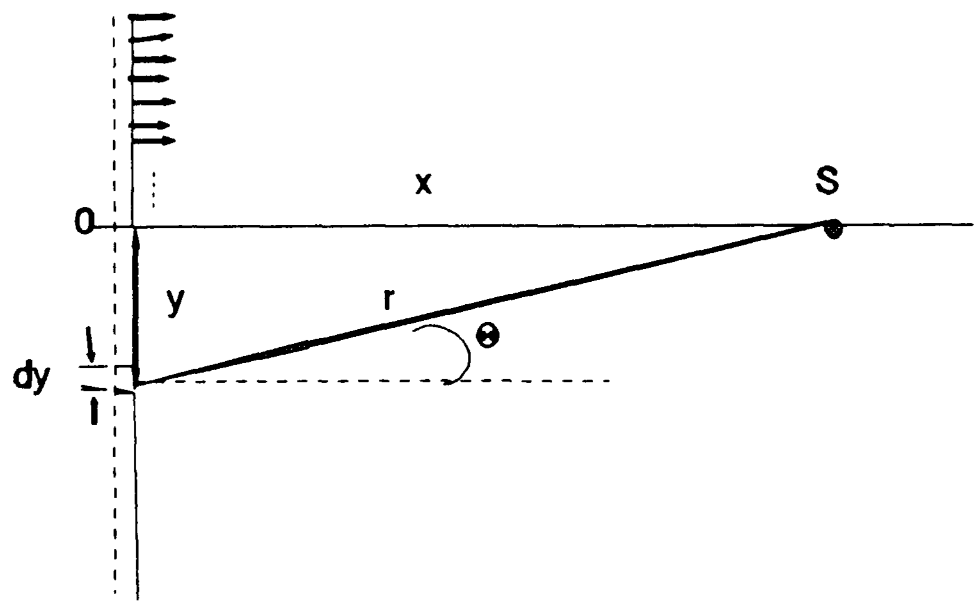


Figure 7-An Infinite Line of Firepower Against a Stationary Point Target

$$dP := 0.5 \cdot (1 + \cos\theta) \cdot e^{-\left[\frac{r}{k}\right]} \cdot \lambda \cdot dy$$

Figure 7 shows that the quantities "y" and θ are completely correlated as are the quantities "r" and θ. Therefore, in order to simplify the equation, one of the variables can be eliminated. We will choose θ. From Figure 3, we see that

$x := r \cdot \cos\theta$ and $y := r \cdot \sin\theta$ .

Solving for cosθ gives

$$\cos\theta := \frac{x}{r} .$$

Substituting t' -rigonometric relationship

$\sin\theta := \sqrt{1 - (\cos\theta)^2}$ into the equation for "y" yields

$$y := r \cdot \sqrt{1 - (\cos\theta)^2} .$$

Substituting for the cosθ term results in

$y := r \cdot \sqrt{1 - \left[\frac{x}{r}\right]^2}$ and

$$y := \sqrt{r^2 - x^2} .$$

Differentiating obtains

$dy := d\left[\sqrt{r^2 - x^2}\right]$ and

$$dy := \left[\frac{r}{\sqrt{r^2 - x^2}}\right] \cdot dr \quad .$$

Substituting for dy and cosθ in the equation for dP gives

$$dP := \left[0.5 \cdot \left[1 + \frac{x}{r}\right] \cdot e^{-\left[\frac{r}{k}\right]} \cdot \lambda \cdot \frac{r}{\sqrt{r^2 - x^2}}\right] \cdot dr \quad .$$

From equation 3-4, the contribution from the upper part of the of the graph, from y = 0 to y = -∞, is

$$P := 0.5 \cdot \lambda \cdot \int_x^\infty \left[1 + \frac{x}{r}\right] \cdot e^{-\left[\frac{r}{k}\right]} \cdot \frac{r}{\sqrt{r^2 - x^2}} \, dr$$

The contribution form the lower part of the graph, from y = 0 to y = +∞, is

$$P := 0.5 \cdot \lambda \cdot \int_x^\infty \left[1 + \frac{x}{r}\right] \cdot e^{-\left[\frac{r}{k}\right]} \cdot \frac{r}{\sqrt{r^2 - x^2}} \, dr$$

The total "P" is then given by the equation

$$P := \left[ 0.5\cdot\lambda\cdot\int_{x}^{\infty}\frac{r+x}{\sqrt{r^{2}-x^{2}}}\cdot e^{-\left[\frac{r}{k}\right]}\,dr \;\ldots \atop + 0.5\cdot\lambda\cdot\int_{\infty}^{x}\frac{r+x}{\sqrt{r^{2}-x^{2}}}\cdot e^{-\left[\frac{r}{k}\right]}\,dr \right] .$$

Because the contributions to "P" from the upper and lower halves of the line are equal, we can write

$$P := \lambda\cdot\int_{x}^{\infty}\frac{r+x}{\sqrt{r^{2}-x^{2}}}\cdot e^{-\left[\frac{r}{k}\right]}\,dr \quad .$$

To solve, we let $r = x*\rho$ and differentiate to obtain $dr = x*d\rho$. Substituting these equations results in

$$P := \lambda\cdot x\cdot\int_{1}^{\infty}\frac{\rho\cdot x+x}{\sqrt{\rho^{2}\cdot x^{2}-x^{2}}}\cdot e^{-\left[\left[\frac{x}{k}\right]\cdot\rho\right]}\,d\rho \quad ,$$

$$P := \lambda \cdot x \cdot \int_1^{\infty} \frac{\rho + 1}{\sqrt{\rho^2 - 1}} \cdot e^{-\left[\left[\frac{x}{k}\right] \cdot \rho\right]} \, d\rho$$ and

$$P := \lambda \cdot x \cdot \int_1^{\infty} \frac{e^{-\left[\left[\frac{x}{k}\right] \cdot \rho\right]}}{\sqrt{\rho^2 - 1}} \cdot \rho \, d\rho + \lambda \cdot x \cdot \int_1^{\infty} \frac{e^{-\left[\left[\frac{x}{k}\right] \cdot \rho\right]}}{\sqrt{\rho^2 - 1}} \, d\rho \quad .$$

The integral on the right is the product of the Bessel function of the first kind of order zero, that is $K_0\left[\frac{x}{k}\right]$ and the and the constants $\lambda$ and "x." Differentiating the Bessel function shows

$$\frac{d\left[K_0\left[\frac{x}{k}\right]\right]}{d\left[\frac{x}{k}\right]} := \frac{d \int_1^{\infty} e^{-\left[\left[\frac{x}{k}\right] \cdot \rho\right]} \, d\rho}{d\left[\frac{x}{k}\right]}$$ and

$$\frac{d\left[K_0\left[\frac{x}{k}\right]\right]}{d\left[\frac{x}{k}\right]} := -\int_1^{\infty} \frac{e^{-\left[\left[\frac{x}{k}\right] \cdot \rho\right]}}{\sqrt{\rho^2 - 1}} \cdot \rho \, d\rho \quad .$$

Substituting this in the integral on the left gives

$$P := -\lambda \cdot x \cdot \left[ \frac{d\left[K_0\left[\frac{x}{k}\right]\right]}{d\left[\frac{x}{k}\right]} \right] + \lambda \cdot x \cdot K_o\left[\frac{x}{k}\right] \quad \text{and}$$

$$P := \lambda \cdot x \cdot \left[ 1 - \frac{d}{d\left[\frac{x}{k}\right]} \right] \cdot K_0\left[\frac{x}{k}\right] \quad .$$

The derivative of the Bessel function of the first kind of order 0 is simply the Bessel function of the second kind of order 1, that is

$K_1\left[\frac{x}{k}\right]$ . This makes our final result

$$P := \lambda \cdot x \cdot \left[ K_0\left[\frac{x}{k}\right] - K_1\left[\frac{x}{k}\right] \right] \quad . \tag{3-5}$$

### C. FINITE LINE OF FIREPOWER AGAINST A STATIONARY POINT TARGET

#### 1. Example 4

Figure 8 shows a finite line of firepower with an orientation parallel to the x-axis and in the positive direction and whose linear charge density has a constant value "λ." We will calculate the magnitude of the field "P" at a point "S" a distance "x" from the line.

Following the calculations for dP as performed in Example 3, we get

$$dP := \left[ 0.5 \cdot \left[ 1 + \frac{x}{r} \right] \cdot e^{-\left[ \frac{r}{k} \right]} \cdot \lambda \cdot \frac{r}{\sqrt{r^2 - x^2}} \right] \cdot dr \quad .$$

From equation 3-4 and the fact that the point "S" is not directly across from the line of firepower, that is that there is no perpendicular intersecting line from the point to the firepower line, we get

$$P := 0.5 \cdot \lambda \cdot \int_{r_1}^{r_2} \left[ 1 + \frac{x}{r} \right] \cdot e^{-\left[ \frac{r}{x} \right]} \cdot \frac{r}{\sqrt{r^2 - x^2}} \, dr \quad ,$$

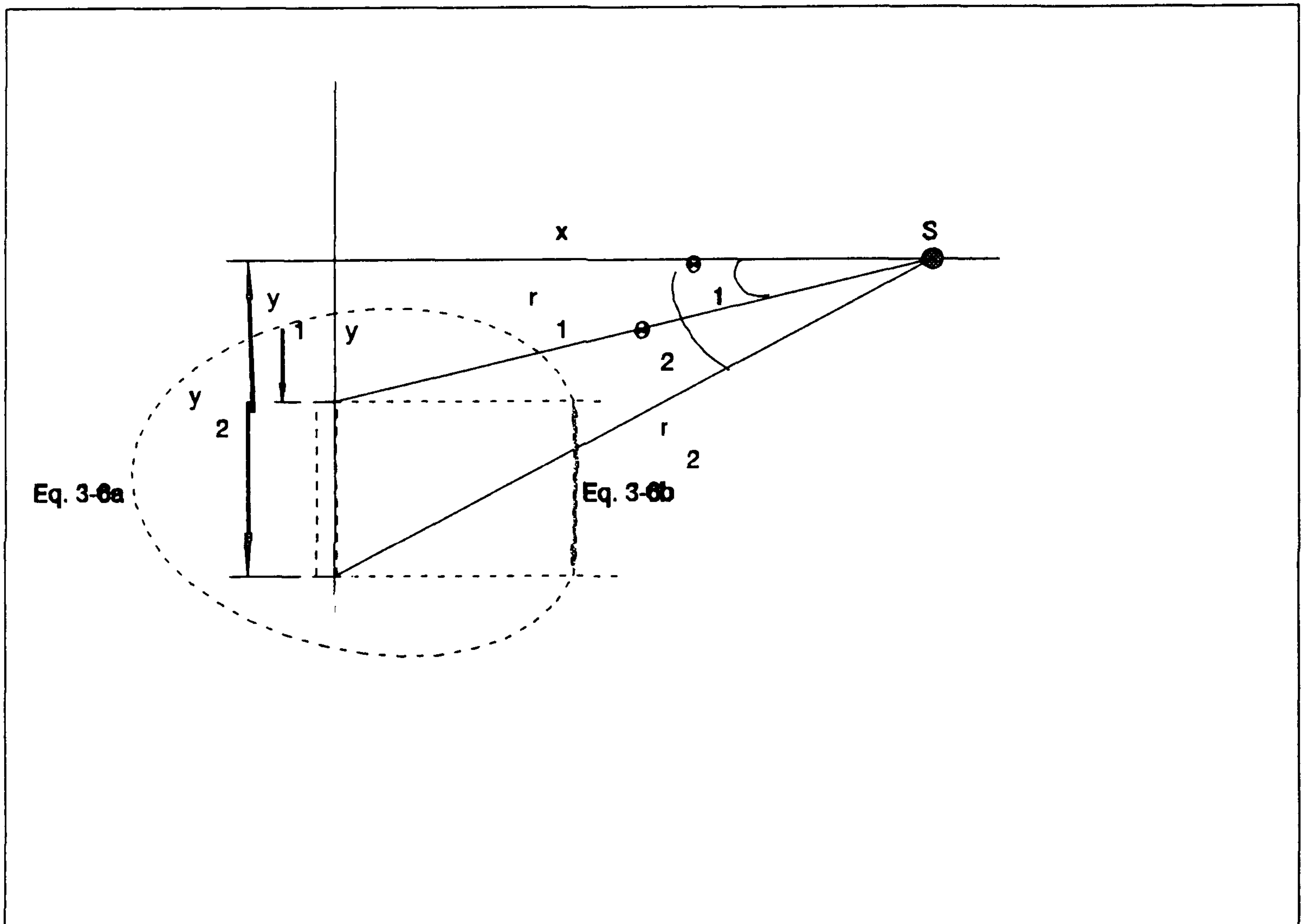


**Figure 8**--A Finite Line of Firepower Against a Stationary Point Target

$$P := 0.5 \cdot \lambda \cdot \int_{r_1}^{r_2} \frac{r + x}{\sqrt{r^2 - x^2}} \cdot e^{-\left[\frac{r}{k}\right]} \, dr \quad \text{and}$$

$$P := \left[ 0.5 \cdot \lambda \cdot \int_{x}^{r_2} \frac{r + x}{\sqrt{r^2 - x^2}} \cdot e^{-\left[\frac{r}{k}\right]} \, dr \; \ldots \; + (-0.5) \cdot \lambda \cdot \int_{x}^{r_1} \frac{r + x}{\sqrt{r^2 - x^2}} \cdot e^{-\left[\frac{r}{k}\right]} \, dr \right]$$

Had the target point been directly across from the line of firepower, the equation above would be the same except that instead of the difference of the two integrals in the equation we would have the sum of the two integrals in the equation, resulting from the change in the limits of integration of the overall problem due to the change in geometry of the targeting situation.

To solve the above equation, we let $r = x*\rho$ and differentiate to obtain $dr = x*d\rho$. Substituting these equations results in

$$P := \left[ 0.5\cdot\lambda\cdot x\cdot \int_{1}^{\rho_2} \frac{\rho\cdot x + x}{\sqrt{\rho^2\cdot x^2 - X^2}}\cdot e^{-\left[\left[\frac{x}{k}\right]\cdot\rho\right]}\,d\rho \;\ldots \atop + (-0.5)\cdot\lambda\cdot x\cdot \int_{1}^{\rho_1} \frac{\rho\cdot x + x}{\sqrt{\rho^2\cdot x^2 - x^2}}\cdot e^{-\left[\left[\frac{x}{k}\right]\cdot\rho\right]}\,d\rho \right]$$

$$P := \left[ 0.5\cdot\lambda\cdot x\cdot \int_{1}^{\rho_2} \frac{\rho + 1}{\sqrt{\rho^2 - 1}}\cdot e^{-\left[\left[\frac{x}{k}\right]\cdot\rho\right]}\,d\rho \;\ldots \atop + (-0.5)\cdot\lambda\cdot x\cdot \int_{1}^{\rho_1} \frac{\rho + 1}{\sqrt{\rho^2 - 1}}\cdot e^{-\left[\left[\frac{x}{k}\right]\cdot\rho\right]}\,d\rho \right]$$

where $\rho_1 = r_1/x$ and $\rho_2 = r_2/x$. Further amplifying the equation for "P" gives

$$P := \begin{bmatrix} 0.5\cdot\lambda\cdot x\cdot\displaystyle\int_1^{\rho_2} \frac{e^{-\left[\left[\frac{x}{k}\right]\cdot\rho\right]}\cdot\rho}{\sqrt{\rho^2-1}}\,d\rho \;\ldots \\ +\,0.5\cdot\lambda\cdot x\cdot\displaystyle\int_1^{\rho_2} \frac{e^{-\left[\left[\frac{x}{k}\right]\cdot\rho\right]}}{\sqrt{\rho^2-1}}\,d\rho \;\ldots \\ +\,(-0.5)\cdot\lambda\cdot x\cdot\displaystyle\int_1^{\rho_1} \frac{e^{-\left[\left[\frac{x}{k}\right]\cdot\rho\right]}\cdot\rho}{\sqrt{\rho^2-1}}\,d\rho \;\ldots \\ +\,(-0.5)\cdot\lambda\cdot x\cdot\displaystyle\int_1^{\rho_1} \frac{e^{-\left[\left[\frac{x}{k}\right]\cdot\rho\right]}}{\sqrt{\rho^2-1}}\,d\rho \end{bmatrix}$$

In order to solve with a software program, let $\rho = 1 + \sigma^2$ and differentiate to obtain $d\rho = 2*\sigma*d\sigma$. Substituting these equations results in

$$
\begin{aligned}
P := \Bigg[\, & 0.5\cdot\lambda\cdot x\cdot\int_{0}^{\sqrt{\rho_{2}-1}} \frac{e^{-\left[\left(\frac{x}{k}\right)\cdot\left(1+\sigma^{2}\right)\right]}\cdot\left(1+\sigma^{2}\right)\cdot 2\cdot\sigma}{\sqrt{\sigma^{4}+2\cdot\sigma^{2}}}\,d\sigma \;\ldots \\
& + 0.5\cdot\lambda\cdot x\cdot\int_{0}^{\sqrt{\rho_{2}-1}} \frac{e^{-\left[\left(\frac{x}{k}\right)\cdot\left(1+\sigma^{2}\right)\right]}\cdot 2\cdot\sigma}{\sqrt{\sigma^{4}+2\cdot\sigma^{2}}}\,d\sigma \;\ldots \\
& + (-0.5)\cdot\lambda\cdot x\cdot\int_{0}^{\sqrt{\rho_{1}-1}} \frac{e^{-\left[\left(\frac{x}{k}\right)\cdot\left(1+\sigma^{2}\right)\right]}\cdot\left(1+\sigma^{2}\right)\cdot 2\cdot\sigma}{\sqrt{\sigma^{4}+2\cdot\sigma^{2}}}\,d\sigma \;\ldots \\
& + (-0.5)\cdot\lambda\cdot x\cdot\int_{0}^{\sqrt{\rho_{1}-1}} \frac{e^{-\left[\left(\frac{x}{k}\right)\cdot\left(1+\sigma^{2}\right)\right]}\cdot 2\cdot\sigma}{\sqrt{\sigma^{4}+2\cdot\sigma^{2}}}\,d\sigma \,\Bigg]
\end{aligned}
$$

$$P := \left[\begin{array}{l} \lambda\cdot x\cdot e^{-\left(\frac{x}{k}\right)}\cdot\displaystyle\int_0^{\sqrt{\rho_2-1}} \frac{e^{-\left[\left(\frac{x}{k}\right)\cdot\sigma^2\right]}\cdot\left(1+\sigma^2\right)}{\sqrt{\sigma^2+2}}\,d\sigma \quad \ldots \\ + \lambda\cdot x\cdot e^{-\left(\frac{x}{k}\right)}\cdot\displaystyle\int_0^{\sqrt{\rho_2-1}} \frac{e^{-\left[\left(\frac{x}{k}\right)\cdot\sigma^2\right]}}{\sqrt{\sigma^2+2}}\,d\sigma \quad \ldots \\ + (-\lambda)\cdot x\cdot e^{-\left(\frac{x}{k}\right)}\cdot\displaystyle\int_0^{\sqrt{\rho_1-1}} \frac{e^{-\left[\left(\frac{x}{k}\right)\cdot\sigma^2\right]}\cdot\left(1+\sigma^2\right)}{\sqrt{\sigma^2+2}}\,d\sigma \quad \ldots \\ + (-\lambda)\cdot x\cdot e^{-\left(\frac{x}{k}\right)}\cdot\displaystyle\int_0^{\sqrt{\rho_1-1}} \frac{e^{-\left[\left(\frac{x}{k}\right)\cdot\sigma^2\right]}}{\sqrt{\sigma^2+2}}\,d\sigma \end{array}\right]$$

$$P := \begin{bmatrix} \lambda \cdot x \cdot e^{-\left(\frac{x}{k}\right)} \cdot \displaystyle\int_0^{\sqrt{\rho_2 - 1}} \frac{e^{-\left[\left(\frac{x}{k}\right)\cdot\sigma^2\right]}}{\sqrt{\sigma^2 + 2}} \cdot \sigma^2 \, d\sigma \ \ldots \\ + 2 \cdot \lambda \cdot x \cdot e^{-\left(\frac{x}{k}\right)} \cdot \displaystyle\int_0^{\sqrt{\rho_2 - 1}} \frac{e^{-\left[\left(\frac{x}{k}\right)\cdot\sigma^2\right]}}{\sqrt{\sigma^2 + 2}} \, d\sigma \ \ldots \\ + (-\lambda) \cdot x \cdot e^{-\left(\frac{x}{k}\right)} \cdot \displaystyle\int_0^{\sqrt{\rho_1 - 1}} \frac{e^{-\left[\left(\frac{x}{k}\right)\cdot\sigma^2\right]}}{\sqrt{\sigma^2 + 2}} \cdot \sigma^2 \, d\sigma \ \ldots \\ + (-2) \cdot \lambda \cdot x \cdot e^{-\left(\frac{x}{k}\right)} \cdot \displaystyle\int_0^{\sqrt{\rho_1 - 1}} \frac{e^{-\left[\left(\frac{x}{k}\right)\cdot\sigma^2\right]}}{\sqrt{\sigma^2 + 2}} \, d\sigma \end{bmatrix} \tag{3-6a}$$

If the target point were directly across from the firepower line, the final solution would be

$$P := \left[\begin{array}{l} \lambda\cdot x\cdot e^{-\left[\frac{x}{k}\right]}\cdot\displaystyle\int_0^{\sqrt{\rho_2-1}} \frac{e^{-\left[\left[\frac{x}{k}\right]\cdot\sigma^2\right]}}{\sqrt{\sigma^2+2}}\cdot\sigma^2\,d\sigma \quad \ldots \\ +\,2\cdot\lambda\cdot x\cdot e^{-\left[\frac{x}{k}\right]}\cdot\displaystyle\int_0^{\sqrt{\rho_2-1}} \frac{e^{-\left[\left[\frac{x}{k}\right]\cdot\sigma^2\right]}}{\sqrt{\sigma^2+2}}\,d\sigma \quad \ldots \\ +\,\lambda\cdot x\cdot e^{-\left[\frac{x}{k}\right]}\cdot\displaystyle\int_0^{\sqrt{\rho_1-1}} \frac{e^{-\left[\left[\frac{x}{k}\right]\cdot\sigma^2\right]}}{\sqrt{\sigma^2+2}}\cdot\sigma^2\,d\sigma \quad \ldots \\ +\,2\cdot\lambda\cdot x\cdot e^{-\left[\frac{x}{k}\right]}\cdot\displaystyle\int_0^{\sqrt{\rho_1-1}} \frac{e^{-\left[\left[\frac{x}{k}\right]\cdot\sigma^2\right]}}{\sqrt{\sigma^2+2}}\,d\sigma \end{array}\right] \qquad (3\text{-}6b)$$

These final two equations are in a format which can be used with mathematics software packages to give answers for finite cases, as will be shown in Example 5.

2. Example 5

Again, using Figure 8, we start with a uniform line of of firepower 200 m long with an orientation parallel to the x-axis and in the positive direction and whose linear charge density has the value 1.40 shots/s*m (the uniform equivalence of distributing 24 riflemen armed with M-16A2's along a 200 m line). Calculate the magnitude of the field of potential firepower "P" at a point "S," offset 100 m to the "left" of the line (in the - y direction) and 200 m forward of the line (in the + x direction). As shown in Figure 8, the values for the variables are $y_1 := 100$ mm, $y_2 := 300$ m and $x := 200$ m. The constant "k" has the value $k := 150$ m, and the linear charge density has the value $\lambda := 1.40$ shots/s*m. From Figure 8, we see that

$$r_1 := \sqrt{y_1^2 + x^2} \text{ meters and } r_2 := \sqrt{y_2^2 + x^2} \text{ meters,}$$

so $r_1 = 223.607$ meters and $r_2 = 360.555$ meters.

From Example 4, we know that

$$\rho_1 := \frac{r_1}{x} \text{ and } \rho_2 := \frac{r_2}{x} \text{ so that}$$

$$\rho_1 = 1.118 \text{ and } \rho_2 = 1.803 \quad .$$

Following the calculations for "P" as performed in Example 4, we get

$$
P := \left[ \begin{array}{l}
\lambda \cdot x \cdot e^{-\left(\frac{x}{k}\right)} \cdot \displaystyle\int_0^{\sqrt{\rho_2 - 1}} \frac{e^{-\left[\left(\frac{x}{k}\right)\cdot \sigma^2\right]}}{\sqrt{\sigma^2 + 2}} \cdot \sigma^2 \, d\sigma \; \ldots \\
+ 2 \cdot \lambda \cdot x \cdot e^{-\left(\frac{x}{k}\right)} \cdot \displaystyle\int_0^{\sqrt{\rho_2 - 1}} \frac{e^{-\left[\left(\frac{x}{k}\right)\cdot \sigma^2\right]}}{\sqrt{\sigma^2 + 2}} \, d\sigma \; \ldots \\
+ (-\lambda) \cdot x \cdot e^{-\left(\frac{x}{k}\right)} \cdot \displaystyle\int_0^{\sqrt{\rho_1 - 1}} \frac{e^{-\left[\left(\frac{x}{k}\right)\cdot \sigma^2\right]}}{\sqrt{\sigma^2 + 2}} \cdot \sigma^2 \, d\sigma \; \ldots \\
+ (-2) \cdot \lambda \cdot x \cdot e^{-\left(\frac{x}{k}\right)} \cdot \displaystyle\int_0^{\sqrt{\rho_1 - 1}} \frac{e^{-\left[\left(\frac{x}{k}\right)\cdot \sigma^2\right]}}{\sqrt{\sigma^2 + 2}} \, d\sigma
\end{array} \right]
$$

Using a mathematics software package the solution can be calculated as $P = 37.503$ effective shots/s.

3. Example 6

Repeat Example 5, but instead of finding the magnitude of the field of potential firepower due to the line of firepower, calculate the field of potential from a single point firepower $q := 11.7$ effective shots/s, concentrated at the midpoint of the line of firepower. Then, calculate the field of potential which results from 24 point firepowers, each of the same magnitude, the sum total of which is the same firepower as in the uniform line of firepower, located at the midpoint of the line of firepower. The values of the variables are $y_m := 200$ m, $x := 200$ m and $k := 150$ m. From Figure 8,

we know that $r_m := \sqrt{y_m^2 + x^2}$, $r_m = 282.843$ meters,

$\cos\theta_m := \frac{x}{r_m}$ and $\cos\theta_m = 0.707$ radians.

From equation 3-1, we get

$$P_m := 0.5 \cdot q \cdot \left[1 + \cos\theta_m\right] \cdot e^{-\left[\frac{r_m}{k}\right]}, \text{ and}$$

$P_m = 1.515$ effective shots/s*shooter.

To calculate the total effective shots/s, if the 24 shooters in the line had the same effective shooting rate, simply multiply the potential from the single point firepower, located at the midpoint, by the 24 shooters to get

$T_m := 24 \cdot P_m$ effective shots/s and

$T_m = 36.368$ effective shots/s.

We note that the solution obtained by concentrating the total firepower in the line at the midpoint is very close to the answer obtained by integrating over the line of firepower to obtain "P," in Example 5. We may state that $P \approx T_m$ and that $T_m$ can, frequently, be used as a good approximation for "P." Note, also, that $T_m$ is slightly less than "P," as one would expect as a result of the exponential relationship of the distance, causing a greater contribution to "P" from the half of the shooting line closest to the target "S" than from the half of the line furthest away from "S."

4. Example 7

Repeat Example 6, but calculate the magnitude of the firepower at each of the endpoints. Then, calculate the total field of potential, if the 24 shooters in the line had the same effective shooting rate. Finally, calculate the average value, $T_{ave}$, of these two concentrated endpoint rates. The values of variables are $x := 200$ m and $k := 150$ m. The distance from the nearest endpoint is $y_n := 100$ m, and the distance from the furthest endpoint is $y_f := 400$ m. From Figure 8, we know that $r_n := \sqrt{y_n^2 + x^2}$, $r_f := \sqrt{y_f^2 + x^2}$, $r_n = 223.607$ m, $r_f = 447.214$ m, $\cos\theta_n := \frac{x}{r_n}$, $\cos\theta_f := \frac{x}{r_f}$, $\cos\theta_n = 0.894$ rad and $\cos\theta_f = 0.447$ rad.

The value of the point firepower is $q := 11.7$ effective shots/s*shooter. From equation 3-1, we get

$$P_n := 0.5 \cdot q \cdot \left[1 + \cos\theta_n\right] \cdot e^{-\left[\frac{r_n}{k}\right]},$$

$$P_f := 0.5 \cdot q \cdot \left[1 + \cos\theta_f\right] \cdot e^{-\left[\frac{r_f}{k}\right]}$$, $P_n = 2.496$ effective shots/s*shooter and $P_f = 0.429$ effective shots/s*shooter.

The totals of effective shots/s, if the 24 shooters had the same effective shooting rates are

$T_n := 24 \cdot P_n$ effective shots/s,

$T_f := 24 \cdot P_f$ effective shots/s,

$T_n = 59.901$ effective shots/s,

$T_f = 10.306$ effective shots/s,

$T_{ave} := \frac{T_n + T_f}{2}$ effective shots/s and

$T_{ave} = 35.104$ effective shots/s.

Note that the $T_m$ approximation for "P" is superior to this approximation of $T_{ave}$, but this example does provide a comparison of the strengths of the two endpoints of the line as a measure of their relative contribution to "P," as calculated in Example 5.

5. Example 8

As shown in Figure 9, we start with a uniform line 200 m long with an orientation parallel to the x-axis and in the positive direction and whose linear charge density has the value 1.40 shots/s*m (the uniform equivalence of distributing 24 riflemen armed with M-16A2's along a 200 m line). Calculate the contribution of 20 meters-long segments of the firepower line towards the total field of potential firepower "P" against a point "S," offset 100 m to the "left" of the line (in the - y direction) and 200 m forward of the line (in the + x direction). We will divide the firepower line into ten equal segments of 20 meters length ( $i := 1 ..10$ ). The coordinates for the endpoints designated by "A" and "B," of the segments are $x := 200$ m, $y_{A_i} :=$ m and $y_{B_i} :=$ m.

| $y_{A_i}$ | $y_{B_i}$ |
|---|---|
| 100 | 120 |
| 120 | 140 |
| 140 | 160 |
| 160 | 180 |
| 180 | 200 |
| 200 | 220 |
| 220 | 240 |
| 240 | 260 |
| 260 | 280 |
| 280 | 300 |

The constant "k" has the value $k := 150$ m, and the linear charge density has the value of $\lambda := 1.40$ shots/s*m, as in as in Example 5. From Figure 9, we see that

$$r_{A_i} := \sqrt{{y_{A_i}}^2 + x^2} \text{ m and } r_{B_i} := \sqrt{{y_{B_i}}^2 + x^2} \text{ m.}$$

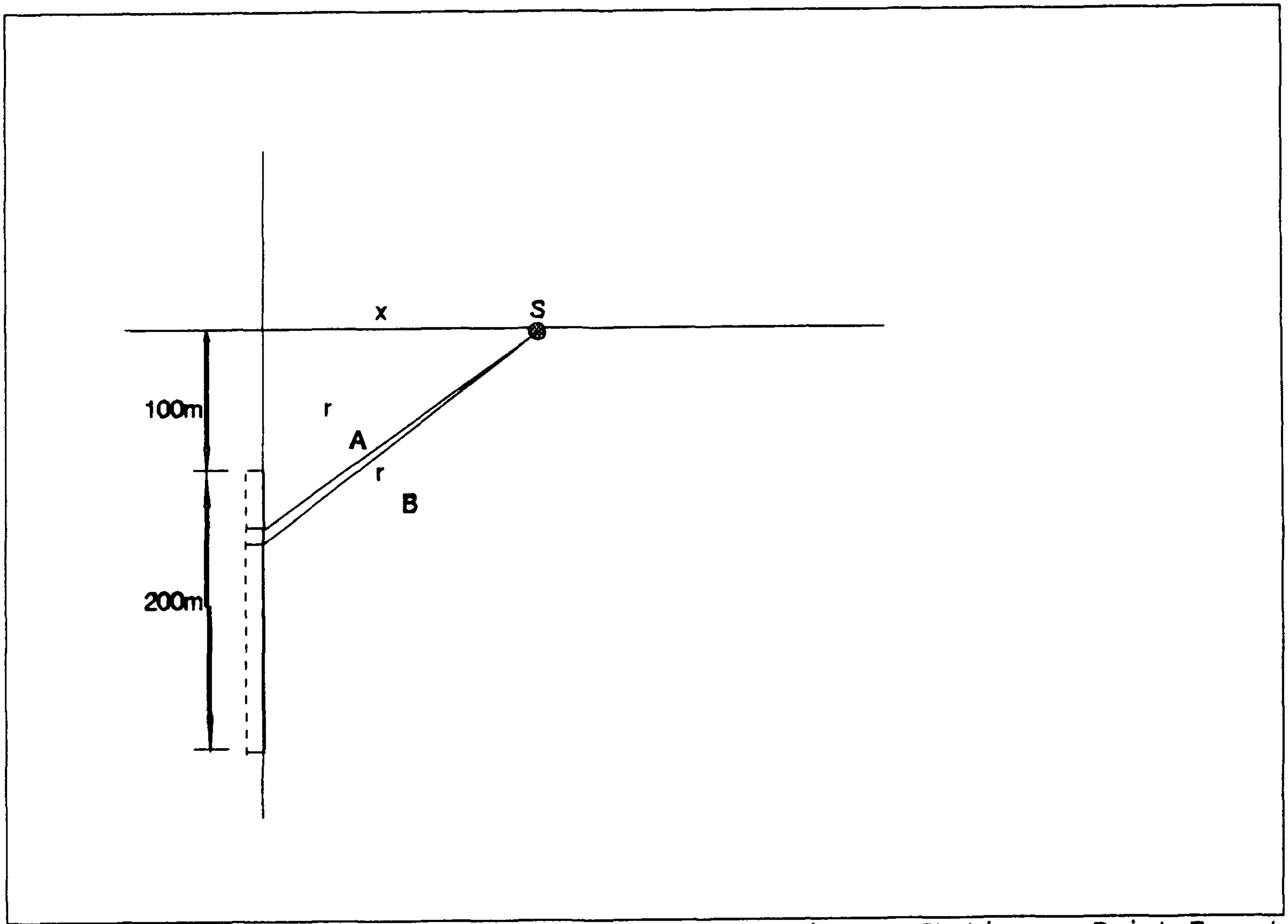

Figure 9--A Segmented Finite Line of Firepower Against a Stationary Point Target

Solving these gives us

$r_{A_i}$ meters and $r_{B_i}$ meters.

| $r_{A_i}$ | $r_{B_i}$ |
|---|---|
| 223.607 | 233.238 |
| 233.238 | 244.131 |
| 244.131 | 256.125 |
| 256.125 | 269.072 |
| 269.072 | 282.843 |
| 282.843 | 297.321 |
| 297.321 | 312.41 |
| 312.41 | 328.024 |
| 328.024 | 344.093 |
| 344.093 | 360.555 |

From Example 4, we know that

$$\rho_{A_i} := \frac{r_{A_i}}{x} \quad \text{and} \quad \rho_{B_i} := \frac{r_{B_i}}{x} \quad .$$

Calculating these results in

$\rho_{A_i}$ and $\rho_{B_i}$ .

| $\rho_{A_i}$ | $\rho_{B_i}$ |
|---|---|
| 1.118 | 1.166 |
| 1.166 | 1.221 |
| 1.221 | 1.281 |
| 1.281 | 1.345 |
| 1.345 | 1.414 |
| 1.414 | 1.487 |
| 1.487 | 1.562 |
| 1.562 | 1.64 |
| 1.64 | 1.72 |
| 1.72 | 1.803 |

Following the procedure used in Example 5, we can solve for the potential field contribution of each segement, as shown below:

$$
P_i := \left[
\begin{aligned}
&\lambda \cdot x \cdot e^{-\left(\frac{x}{k}\right)} \cdot \int_0^{\sqrt{\rho_{B_i}-1}} \frac{e^{-\left[\left(\frac{x}{k}\right)\cdot\sigma^2\right]}}{\sqrt{\sigma^2+2}} \cdot \sigma^2 \, d\sigma \; \ldots \\
&+ 2\cdot\lambda \cdot x \cdot e^{-\left(\frac{x}{k}\right)} \cdot \int_0^{\sqrt{\rho_{B_i}-1}} \frac{e^{-\left[\left(\frac{x}{k}\right)\cdot\sigma^2\right]}}{\sqrt{\sigma^2+2}} \, d\sigma \; \ldots \\
&+ (-\lambda)\cdot x \cdot e^{-\left(\frac{x}{k}\right)} \cdot \int_0^{\sqrt{\rho_{A_i}-1}} \frac{e^{-\left[\left(\frac{x}{k}\right)\cdot\sigma^2\right]}}{\sqrt{\sigma^2+2}} \cdot \sigma^2 \, d\sigma \; \ldots \\
&+ (-2)\cdot\lambda \cdot x \cdot e^{-\left(\frac{x}{k}\right)} \cdot \int_0^{\sqrt{\rho_{A_i}-1}} \frac{e^{-\left[\left(\frac{x}{k}\right)\cdot\sigma^2\right]}}{\sqrt{\sigma^2+2}} \, d\sigma
\end{aligned}
\right]
$$

Using a mathematics software package, the solution, with the segments nearest the target shown at the top, can be calculated as $P_i$ effective shots/s.

| $P_i$ |
|---|
| 5.734 |
| 5.247 |
| 4.76 |
| 4.288 |
| 3.84 |
| 3.424 |
| 3.041 |
| 2.693 |
| 2.379 |
| 2.098 |

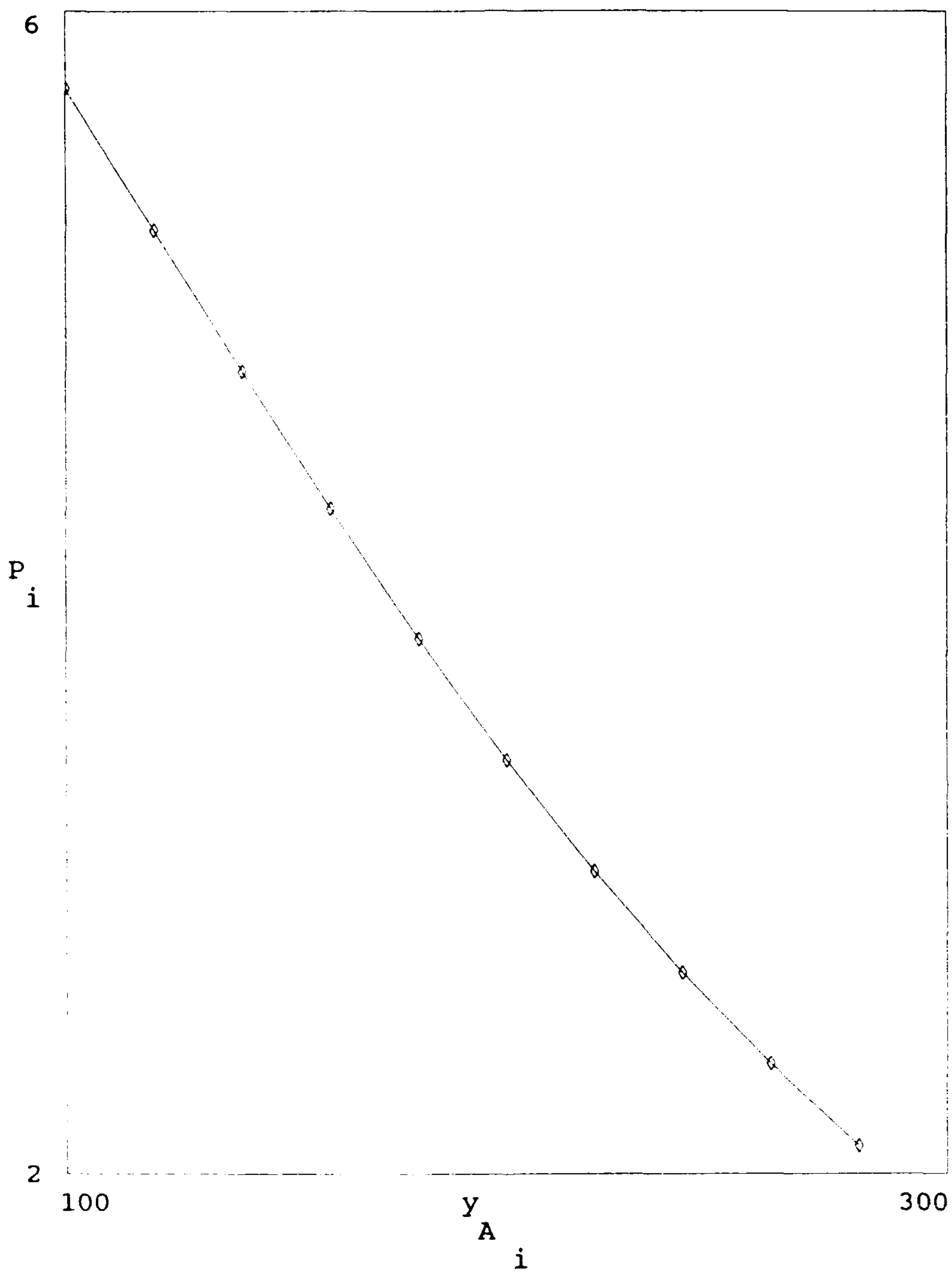


Figure 10--Plot of Magnitude of Potential Firepower, $P_i$, (effective shots per second) Versus Range, $y_{A_i}$, (meters).

## D. A FINITE LINE OF FIREPOWER AGAINST A STATIONARY LINE TARGET

### 1. Example 9

As shown in Figure 11, we start with a uniform line 200 m long with an orientation parallel to the x-axis and in the positive direction and whose linear charge density has the value 1.40 shots/s*m (the uniform equivalence of distributing 24 riflemen armed with M-16A2's along a 200 m line). Calculate the magnitude of the field of potential firepower "P" against a 100 m enemy front at various points along the front, which is centered about a point "M," with endpoints "A" and "B" and offset 100 m to the "left" of the line (in the - y direction) and 200 m forward of the line (in the + x direction). We will calculate the field at ten one-meter-long segments along the target line, $i := 1 \,..10$ . The coordinates for the endpoints, designated by "A" and "B," of the segments are

$x := 200$ m, $\quad y_{A_i} :=$ m and $\quad y_{B_i} :=$ m.

| $y_{A_i}$ | $y_{B_i}$ |
|---|---|
| 50.5 | 250.5 |
| 60.5 | 260.5 |
| 70.5 | 270.5 |
| 80.5 | 280.5 |
| 90.5 | 290.5 |
| 100.5 | 300.5 |
| 110.5 | 310.5 |
| 120.5 | 320.5 |
| 130.5 | 330.5 |
| 140.5 | 340.5 |

The constant "k" has the value $k := 150$ m, and the linear charge density has the value of 1.40 shots/s*m. Assuming the targets on the enemy line are uniformly distributed, each one

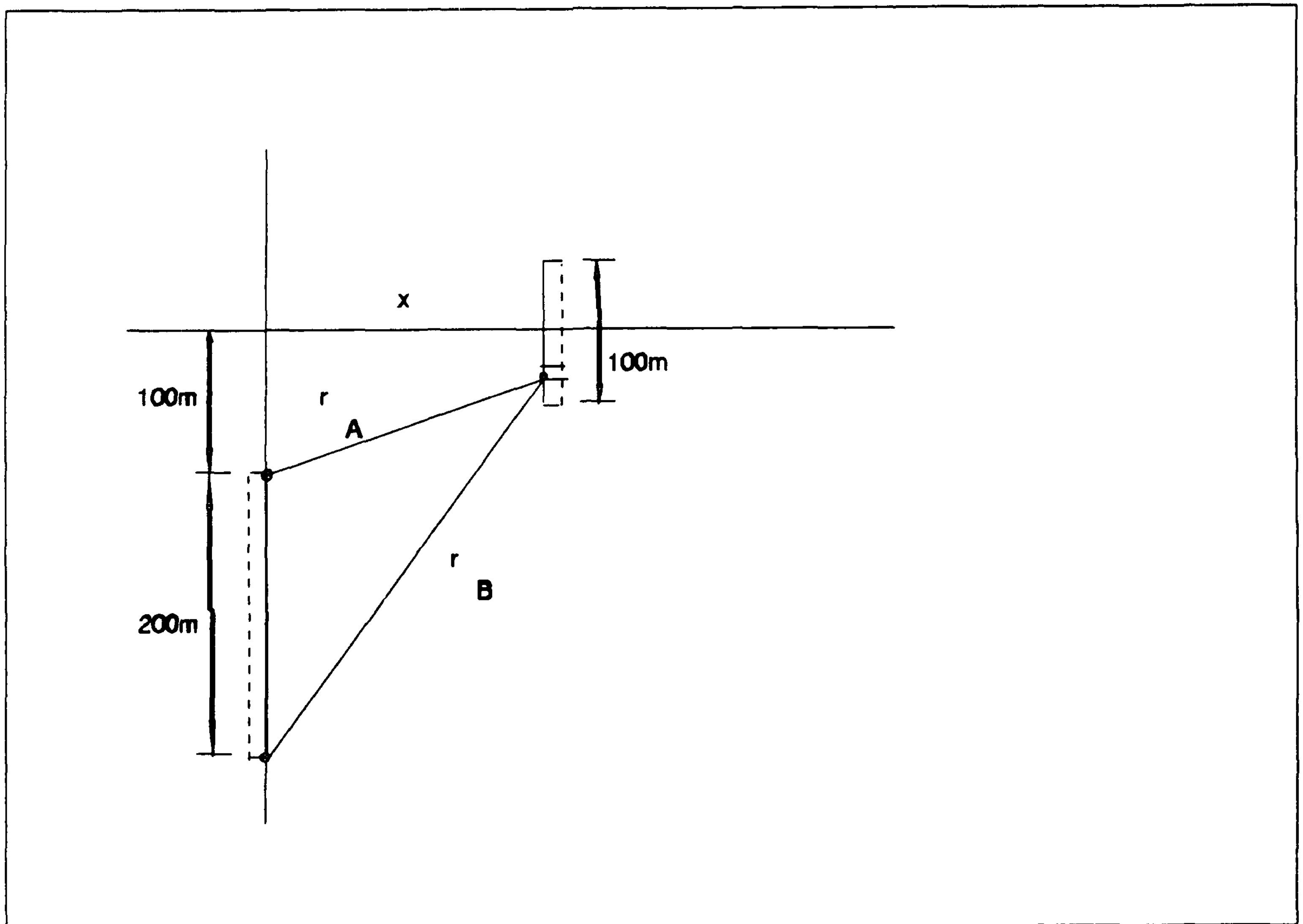


Figure 11--A Finite Line of Firepower Against a Stationary Point Target

meter segment will only receive one one-hundredth of the total effect. As a result, the equivalent linear charge density is $\lambda := 0.014$ shots/s*m for each one meter of target segment. From Figure 10, we see that

$$r_{A_i} := \sqrt{{y_{A_i}}^2 + x^2}\ \text{m and}\quad r_{B_i} := \sqrt{{y_{B_i}}^2 + x^2}\ \text{m,}\quad \text{and}$$

$r_{A_i}$ meters and $r_{B_i}$ meters.

| $r_{A_i}$ | $r_{B_i}$ |
|---|---|
| 206.277 | 320.547 |
| 208.95 | 328.421 |
| 212.062 | 336.408 |
| 215.593 | 344.5 |
| 219.523 | 352.69 |
| 223.831 | 360.971 |
| 228.496 | 369.338 |
| 233.496 | 377.783 |
| 238.81 | 386.303 |
| 244.418 | 394.893 |

From Example 4, we know that

$$\rho_{A_i} := \frac{r_{A_i}}{x} \quad \text{and} \quad \rho_{B_i} := \frac{r_{B_i}}{x}.$$

Calculating these results in

$\rho_{A_i}$ and $\rho_{B_i}$.

| $\rho_{A_i}$ | $\rho_{B_i}$ |
|---|---|
| 1.031 | 1.603 |
| 1.045 | 1.642 |
| 1.06 | 1.682 |
| 1.078 | 1.723 |
| 1.098 | 1.763 |
| 1.119 | 1.805 |
| 1.142 | 1.847 |
| 1.167 | 1.889 |
| 1.194 | 1.932 |
| 1.222 | 1.974 |

Following the procedure used in Example 5, we can solve for the potential field of each segment of the enemy line, as shown below:

$$P_i := \left[\begin{array}{l} \lambda \cdot x \cdot e^{-\left[\frac{x}{k}\right]} \cdot \displaystyle\int_0^{\sqrt{\rho_{B_i} - 1}} \frac{e^{-\left[\left[\frac{x}{k}\right] \cdot \sigma^2\right]}}{\sqrt{\sigma^2 + 2}} \cdot \sigma^2 \, d\sigma \; \ldots \\ + 2 \cdot \lambda \cdot x \cdot e^{-\left[\frac{x}{k}\right]} \cdot \displaystyle\int_0^{\sqrt{\rho_{B_i} - 1}} \frac{e^{-\left[\left[\frac{x}{k}\right] \cdot \sigma^2\right]}}{\sqrt{\sigma^2 + 2}} \, d\sigma \; \ldots \\ + (-\lambda) \cdot x \cdot e^{-\left[\frac{x}{k}\right]} \cdot \displaystyle\int_0^{\sqrt{\rho_{A_i} - 1}} \frac{e^{-\left[\left[\frac{x}{k}\right] \cdot \sigma^2\right]}}{\sqrt{\sigma^2 + 2}} \cdot \sigma^2 \, d\sigma \; \ldots \\ + (-2) \cdot \lambda \cdot x \cdot e^{-\left[\frac{x}{k}\right]} \cdot \displaystyle\int_0^{\sqrt{\rho_{A_i} - 1}} \frac{e^{-\left[\left[\frac{x}{k}\right] \cdot \sigma^2\right]}}{\sqrt{\sigma^2 + 2}} \, d\sigma \end{array}\right]$$

Using a mathematics software package, the solution, in the respective one meter segements, at ten meter intervals, with the nearest segments at the top, can be found as

$P_i$ effective shots/s.

| $P_i$ |
|---|
| 0.479 |
| 0.457 |
| 0.436 |
| 0.415 |
| 0.394 |
| 0.374 |
| 0.354 |
| 0.335 |
| 0.317 |
| 0.299 |

2. Example 10

Repeat Example 9, but divide the enemy line into ten segments of ten meters length, and calculate the potential firepower being directed against each of these segments. We will separate the line into ten segments ( $i := 1 \,..10$ ). The midpoints of the segments will be used in calculating the answer. It will be assumed that the line of firepower is firing uniformly at the enemy line. If every one meter receives the effect of one one-hundredth of the total linear firepower density, then each ten meter long segments should receive the effect of one-tenth of the total linear firepower density, integrated over the line of firepower towards the midpoint of the ten meter long segments of the enemy line. The distances from the endpoints, designated by "A" and "B," to the midpoints of the ten meter long segments are

$x := 200$ m, $\quad y_{A_i} :=$ m and $\quad y_{B_i} :=$ m.

| $y_{A_i}$ | $y_{B_i}$ |
|---|---|
| 55 | 255 |
| 65 | 265 |
| 75 | 275 |
| 85 | 285 |
| 95 | 295 |
| 105 | 305 |
| 115 | 315 |
| 125 | 325 |
| 135 | 335 |
| 145 | 345 |

The constant "k" has the value $k := 150$ m, and the linear charge density has the value of 1.40 shots/s*m. Each ten meter segment, however, assuming the line of firepower targets the enemy line uniformly, will only receive one one-tenth of the total effect, resulting in an equivalent linear charge density of $\lambda := 0.14$ shots/s*m for each one meter segment. From Figure 11, we see that

$$r_{A_i} := \sqrt{{y_{A_i}}^2 + x^2}\ \text{m and}\quad r_{B_i} := \sqrt{{y_{B_i}}^2 + x^2}\ \text{m.}$$

Solving these gives us

$r_{A_i}$ meters and $r_{B_i}$ meters.

| $r_{A_i}$ | $r_{B_i}$ |
|---|---|
| 207.425 | 324.076 |
| 210.297 | 332.002 |
| 213.6 | 340.037 |
| 217.313 | 348.174 |
| 221.416 | 356.406 |
| 225.887 | 364.726 |
| 230.705 | 373.129 |
| 235.85 | 381.608 |
| 241.299 | 390.16 |
| 247.032 | 398.779 |

From Example 4, we know that

$$\rho_{A_i} := \frac{r_{A_i}}{x} \text{ and } \rho_{B_i} := \frac{r_{B_i}}{x}.$$

Calculating these results in

$\rho_{A_i}$ and $\rho_{B_i}$.

| $\rho_{A_i}$ | $\rho_{B_i}$ |
|---|---|
| 1.037 | 1.62 |
| 1.051 | 1.66 |
| 1.068 | 1.7 |
| 1.087 | 1.741 |
| 1.107 | 1.782 |
| 1.129 | 1.824 |
| 1.154 | 1.866 |
| 1.179 | 1.908 |
| 1.206 | 1.951 |
| 1.235 | 1.994 |

Following the procedure used in Example 5, we can solve for the potential field of each segment of the enemy line, as shown below:

$$P_i := \left[\begin{aligned}
&\lambda\cdot x\cdot e^{-\left(\frac{x}{k}\right)}\cdot\int_0^{\sqrt{\rho_{B_i}-1}}\frac{e^{-\left[\left(\frac{x}{k}\right)\cdot\sigma^2\right]}}{\sqrt{\sigma^2+2}}\cdot\sigma^2\,d\sigma\;\ldots\\
&+2\cdot\lambda\cdot x\cdot e^{-\left(\frac{x}{k}\right)}\cdot\int_0^{\sqrt{\rho_{B_i}-1}}\frac{e^{-\left[\left(\frac{x}{k}\right)\cdot\sigma^2\right]}}{\sqrt{\sigma^2+2}}\,d\sigma\;\ldots\\
&+(-\lambda)\cdot x\cdot e^{-\left(\frac{x}{k}\right)}\cdot\int_0^{\sqrt{\rho_{A_i}-1}}\frac{e^{-\left[\left(\frac{x}{k}\right)\cdot\sigma^2\right]}}{\sqrt{\sigma^2+2}}\cdot\sigma^2\,d\sigma\;\ldots\\
&+(-2)\cdot\lambda\cdot x\cdot e^{-\left(\frac{x}{k}\right)}\cdot\int_0^{\sqrt{\rho_{A_i}-1}}\frac{e^{-\left[\left(\frac{x}{k}\right)\cdot\sigma^2\right]}}{\sqrt{\sigma^2+2}}\,d\sigma
\end{aligned}\right]$$

Using a mathematics software package, the solution, in ten meter target segments with the nearest segments at the top, can be found as $P_i$ effective shots/s.

| $P_i$ |
|---|
| 4.692 |
| 4.478 |
| 4.266 |
| 4.056 |
| 3.851 |
| 3.651 |
| 3.456 |
| 3.268 |
| 3.087 |
| 2.913 |

## E. EXAMPLE 11: A FINITE LINE OF FIREPOWER AGAINST A MOVING POINT TARGET

As shown in Figure 12, we start with a uniform line 200 m long with an orientation parallel to the x-axis and in the positive direction and whose linear charge density has the value 1.40 shots/s*m. We calculate the potential effective shots absorbed by a moving point target "T," moving at a constant speed over a known path. Then, we calculate the potential shots per second at the initial and final positions of the target and compare the two. Then, we compute how the answer changes if the speed of the target is doubled.

To solve the problem, we need to integrate over the path, in order to find the cumulative potential firepower, "P" in effective shots per second, applied to the point while moving over the length of the path, and then divide by the speed of the point target, $s := 4.00$ meters per second. The constant "k" has the value $k := 150$ m and $\lambda := 1.40$ m. The motion of "T" is the quarter-circle, with a radius $r := 400$ meters described by the equations $x(t) := r \cdot \cos(t)$ and $y(t) := r \cdot \sin(t)$ taken over the range of minus one-half pi to zero, where the center of the coordinate system is in the middle of the shooting line, and $L := 100$ m, as shown in Figure 12. The equations for the distances from the endpoints the line to the target are

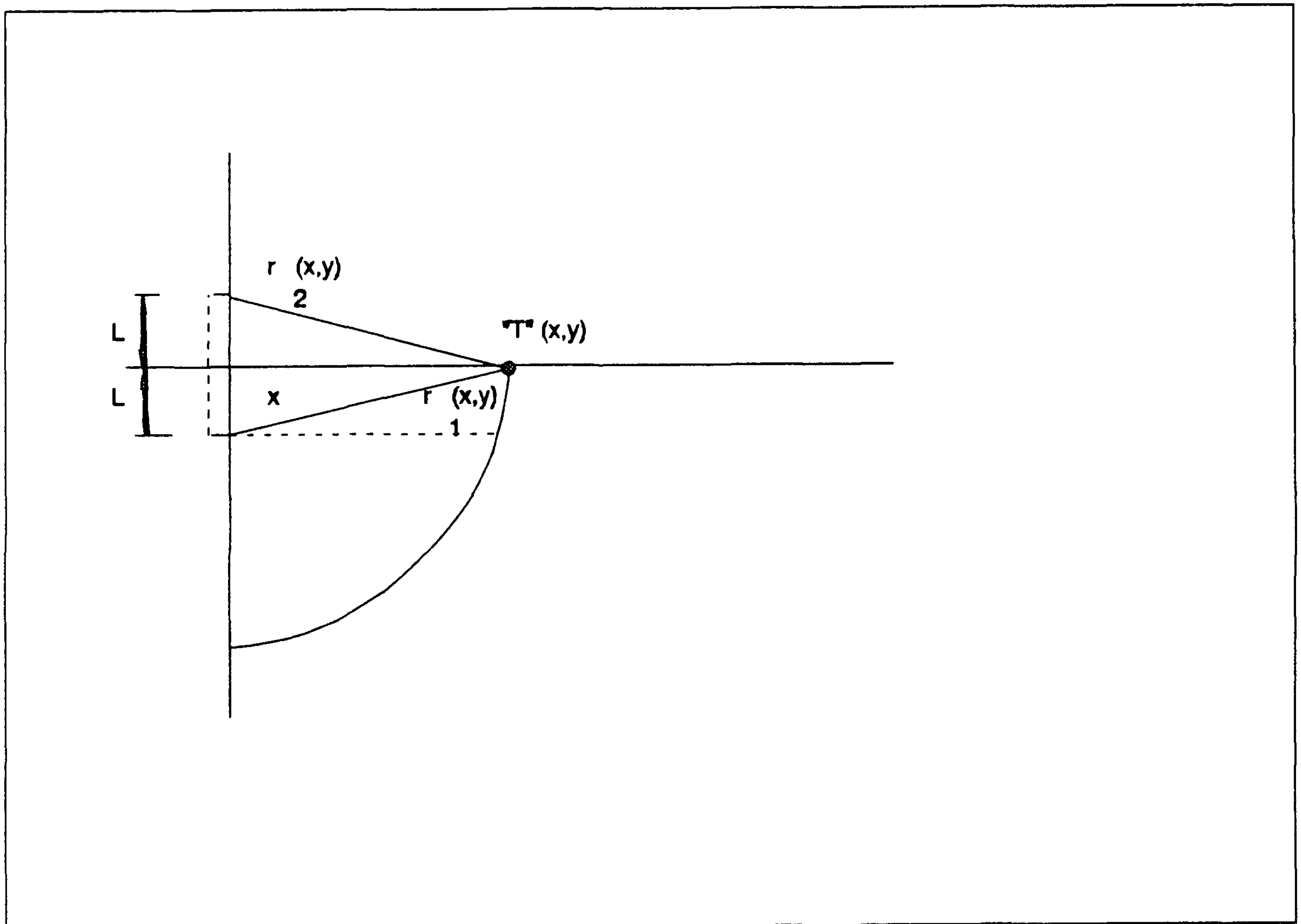


Figure 12--A Finite Line of Firepower Against a Moving Point Target

$r_1(x,y) := \sqrt{x^2 + (y + L)^2}$ m and

$r_2(x,y) := \sqrt{x^2 + (y - L)^2}$ m. From Example 4, we know that

$$\rho_1(x,y) := \frac{r_1(x,y)}{x} \quad \text{and} \quad \rho_2(x,y) := \frac{r_2(x,y)}{x} .$$

The formula for calculating the potential field at any point along the path of target "T," which is directly across from the line of firepower, is shown below:

$$
Q(x,y) := \left[
\begin{aligned}
&\lambda \cdot x \cdot e^{-\left[\frac{x}{k}\right]} \cdot \int_{0}^{\sqrt{\rho_{2}(x,y) - 1}} \frac{e^{-\left[\left[\frac{x}{k}\right] \cdot \sigma^{2}\right]}}{\sqrt{\sigma^{2} + 2}} \cdot \sigma^{2} \, d\sigma \ \ldots \\
&+ 2 \cdot \lambda \cdot x \cdot e^{-\left[\frac{x}{k}\right]} \cdot \int_{0}^{\sqrt{\rho_{2}(x,y) - 1}} \frac{e^{-\left[\left[\frac{x}{k}\right] \cdot \sigma^{2}\right]}}{\sqrt{\sigma^{2} + 2}} \, d\sigma \ \ldots \\
&+ \lambda \cdot x \cdot e^{-\left[\frac{x}{k}\right]} \cdot \int_{0}^{\sqrt{\rho_{1}(x,y) - 1}} \frac{e^{-\left[\left[\frac{x}{k}\right] \cdot \sigma^{2}\right]}}{\sqrt{\sigma^{2} + 2}} \cdot \sigma^{2} \, d\sigma \ \ldots \\
&+ 2 \cdot \lambda \cdot x \cdot e^{-\left[\frac{x}{k}\right]} \cdot \int_{0}^{\sqrt{\rho_{1}(x,y) - 1}} \frac{e^{-\left[\left[\frac{x}{k}\right] \cdot \sigma^{2}\right]}}{\sqrt{\sigma^{2} + 2}} \, d\sigma
\end{aligned}
\right]
$$

The formula for calculating the potential field at any point along the path of target "T," which is not directly across from the line of firepower, is shown below:

$$P(x,y) := \left[ \begin{aligned} &\lambda \cdot x \cdot e^{-\left[\frac{x}{k}\right]} \cdot \int_0^{\sqrt{\rho_2(x,y) - 1}} \frac{e^{-\left[\left[\frac{x}{k}\right]\cdot\sigma^2\right]}}{\sqrt{\sigma^2 + 2}} \cdot \sigma^2 \, d\sigma \quad \ldots \\ &+ 2 \cdot \lambda \cdot x \cdot e^{-\left[\frac{x}{k}\right]} \cdot \int_0^{\sqrt{\rho_2(x,y) - 1}} \frac{e^{-\left[\left[\frac{x}{k}\right]\cdot\sigma^2\right]}}{\sqrt{\sigma^2 + 2}} \, d\sigma \quad \ldots \\ &+ (-\lambda) \cdot x \cdot e^{-\left[\frac{x}{k}\right]} \cdot \int_0^{\sqrt{\rho_1(x,y) - 1}} \frac{e^{-\left[\left[\frac{x}{k}\right]\cdot\sigma^2\right]}}{\sqrt{\sigma^2 + 2}} \cdot \sigma^2 \, d\sigma \quad \ldots \\ &+ (-2) \cdot \lambda \cdot x \cdot e^{-\left[\frac{x}{k}\right]} \cdot \int_0^{\sqrt{\rho_1(x,y) - 1}} \frac{e^{-\left[\left[\frac{x}{k}\right]\cdot\sigma^2\right]}}{\sqrt{\sigma^2 + 2}} \, d\sigma \end{aligned} \right]$$

The total shots received by "T" is given by TOL := 1

$$F := \frac{1}{s} \cdot \int_{-0.253}^{0} Q(x(t),y(t))\,dt \;\ldots$$

$$+ \frac{1}{s} \cdot \int_{\frac{-\pi}{2}}^{-0.253} P(x(t),y(t))\,dt \qquad (3\text{-}7)$$

where $\theta = -0.253$ radians is the point along the path of "T" at which "T" goes from being directly in front of the firing line to being on a flank, and we get $F = 6.401$ effective shots.

The potential rate at the initial point is $Q(400,0) = 18.839$ effective shots/s, and the potential rate at the final point is $P(0.0001,-400) = 10.465$ effective shots/s. The advantage in maneuvering to the enemy's flank is obvious--a diminishment in effective firepower rates equivalent to a 45% reduction. The advantage of moving quickly, also, is evident. Moving twice as fast means a one-half reduction in effective shots. If moving from one covered position to another covered position, this shows the advantage in being able to move quickly.

One must also note the restricion on equation 3-6a: the equation will not calculate a value for "P" if $x = 0$. The restriction is easily bypassed by using a value such as the one used above, $x = 0.0001$, and the resulting error in this approximation of zero is insignificant.

# IV. CONCLUSIONS/RECOMMENDATIONS

## A. SUMMARY OF MODEL THEORY

The model evolved from the premise that an accurate estimate of the dynamic changes of combat power of two forces on a field of battle necessitates identifying the spatial relationship between the two forces. From this dependent relationship, force strengths could then be calculated. Proceeding from this relativity concept, an identification of the change, with time, in the spatial relationship and the corresponding change of force strengths would result in a measure of the value of maneuver in combat. This caliber of mobility would not be based upon the dimensional units of mobility, such as meters per second, but rather on the change in the relative force strengths resulting from movement of the forces over a period of time. In order to embody these concepts in a model, the model would have to measure the relative combat power of a force and allow an evaluation of the dynamic spatial changes of a force. Following this approach, the model was developed as an attempt to describe the dynamics of fire analytically and incorporate the value of movement into a ground combat model, which would describe the intensity in a field of fire.

## B. SUMMARY OF MODEL DEVELOPMENT

Effective shots per second at the target were chosen as the units of measure of the model. The development limited the scope of the model to ground combat with small arms. Two of four phenomena showing the effect which movement has upon targeting were chosen to be described in the model. Modeling one of the phenomena involved developing a range function which describes how movement which decreases the range to a target causes an increase in the targeting effectiveness. Modeling the other phenomena involved developing an orientation function which describes how movement away from the direction from which a defender expects and prepares for an attack will cause a decrease in the effectiveness of the defender. The range factor incorporated into the model was chosen as an exponential decay with range, dampened by a parameter equal to one-fourth of the maximum effective range of the respective weapon. The orientation factor incorporated into the model was chosen to be a cardioid effect, with the maximum effectiveness directly in front of a shooter and diminishing accordingly as one moves around the flanks. The final model was determined to be

$$P = 0.5 * q * (1 + \cos\theta) * e^{-r/k} .$$

The variable "q" represents the maximum effective firing rate; "r" represents the range; $\theta$ represents the angle offset from the shooter's orientation axis, and "k" is a constant equal to one-fourth the maximum range.

## C. CONCLUSIONS

Our premise that identifying the spatial relationships between two forces on a field of battle was required in order to accurately estimate the dynamic changes of combat power appears to be correct. This can be seen in all of the examples, each of which displays that the position of an opposing force relative to one's position and orientation determines one's combat power. From Example 11, we can conclude that the value of mobility can be determined from our combat model, which measures combat power in terms of effective shots per second. Equation 3-7, allows for a dynamic comparison of combat power based upon different speeds and paths of movement of the enemy target.

The model, in the form developed above, can be used as a decision aid by a commander to evaluate tactical options in approaching an enemy. The commander may evaluate different speeds, different directions and different troop placements with the model. Uses of the model can be increased significantly when other factors, e.g. suppression of fire, suppression of movement, terrain factors, weather factors, etc., are included. In expanding the versatility of the model with these factors, one must first justify the values of these additional variables. In most cases, data must be collected and analyzed prior to adding any of these variables to a quantitative model.

In making use of the model, one must keep in mind the difference in computational procedures between the case in which the target is directly across from the firing line and the case in which the target is to either side of the firing line. *The midpoint approximation of a line, as shown in* Example 6, works well except when the target point lies directly across from the line, as opposed to being on one side or the other of the line. For this exception, one must draw a perpendicular line from the target point to the firing line, separating the line into two segments; identify the largest of the two line segments, and use the midpoint of this segment for the midpoint approximation. The result is still a highly accurate approximation of the integrals used in the continuous case.

When performing the integrals in the continuous case, using equation 3-6a, when the target is on either side of the firing line, or equation 3-6b, when the target is directly across from the firing line, as developed in Example 4, one has to identify when to use each equation. This problem is simple for all situations involving stationary targets. In cases involving a moving target, one must use the procedures performed in Example 11 to ensure that the limits of integration in the integrals allow for the correct integration across the entire line of firepower, no more and no less.

## D. FUTURE DEVELOPMENT

Future development of the model should concentrate on the use of points and lines to accurately approximate the dispositions and movements of actual combat forces. Future evaluation of the model should include developing further realistic examples involving moving forces. With the support of appropriate data a suppression factor could be included, which would, then, allow for an examination of decisions between having combat assets move versus having combat assets maintain their positions and fire back at the enemy. A model displaying the tradeoffs between possible options would be of great value. Hopefully, an examination of this model will guide data collectors in collecting data to validate, modify and expand this model. An important factor which needs to be included in an overall model is a factor representing the speed with which a firing unit can realign its axis of orientation, in order to allow the model to perform time lapse comparison of possible strategies.

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